



Mathematical Modelling of River Pollution as a Result of Pipeline Damage

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A mathematical model of the process of pollution of the aquatic environment with oil products at the rupture of an oil pipeline crossing a river bed is presented. To describe this process, Reynolds equations are used for turbulent flow. The discrete analogue is obtained using the finite volume method. As a result of the numerical solution, the influence of the river parameters and the characteristics of the source of pollution on the process of distribution of petroleum products in the aquatic environment were studied. The results of mathematical modelling of oil spills can be used in the development of preventive measures to detect and prevent the spread of pollution.

1. Introduction

The first studies simulated the spread of petroleum products in calm waters. The oil slick was considered to be round. In this case, only an increase in its diameter was considered (Buckmaster, 1973). These problems were solved in one-dimensional or axisymmetric formulations. Later, models were developed that more realistically simulated the processes of the spread of oil products in the aquatic environment. They took into account the processes caused not only by the action of wind and sea currents, but also evaporation, dissolution, emulsification, etc. The literature contains data on mathematical modelling of water pollution by oil products, but they mainly relate to the study of pollution in the seas (Li et al., 2007). The study of Guandalini et al. (2017) described the development of a methodology to evaluate the dynamics related to the dispersion of hydrocarbons in the sea. In contrast to previous paper using two-dimensional approach (Issakhov et al., 2021), the spatial case is considered in this paper. It should be noted that most of the papers on the pollution of the aquatic environment with oil products is mainly devoted to either modeling the flow of oil in pipes and assessing damage in case of rupture of oil pipelines (Sun, 2012), or accidental pollution of the seas from ships. In paper (Xu et al. (2019)), an attempt is made to study the process of oil products outflow from the back surface of an underwater blunt body in the sea in a two-dimensional setting. Unfortunately, this paper does not contain data on the further spread of oil products in the aquatic environment. In the presented paper, we consider a section of a river with a given flow rate, at the bottom of which a pipeline is located in the underwater crossing system. The situation is considered when damage to the pipeline occurs and oil products enter the open water body. The processes occurring in the water when petroleum products get into it are influenced by: wind speed and direction, flow velocity and river geometry, as well as damage characteristics and power output. Oil is able to evaporate, dissolve, biodegrade, photo- and thermochemically decompose, emulsify with water, settle at the bottom and float to the surface, adsorb and absorb, envelop suspended particles, and also interact with biological organisms. The study area is considered to be the part of the river near the pipeline rupture.

2. Mathematical and Physical Setting

In this problem, a section of a river with a given flow rate is considered, at the bottom of which there is a pipeline in the underwater crossing system. The situation is considered when the pipeline is damaged and oil

products enter the open water body. The processes occurring in water when oil products get into it are influenced by: wind speed and direction, flow speed and geometry of the river, as well as the characteristics of damage and the power of the release. Oil can evaporate, dissolve, biodegrade, photo- and thermochemically, emulsify with water, settle at the bottom and float to the surface, adsorb and be absorbed, envelop suspended particles, and also interact with biological organisms. A part of the river near the rupture of the oil pipeline is considered as a research area. At the lower boundary of the computational domain, the parameters of the pollution source formed as a result of damage to the oil pipeline are set (Figure 1). The direction of the flow of the river coincides with the direction of the x_1 axis, and the flow rate from the intended opening of the pipeline with the direction of the x_3 axis. The purpose of this paper is to obtain the distribution of fields of concentration of petroleum products flowing from the pipeline, temperature and velocity, as well as to determine the distribution area of pollutants to predict this emergency situation and its subsequent elimination. A section of the river was considered in the form of a parallelepiped of size (6m × 4m × 2m). The coordinates and surface areas of the source of pollution at the bottom of the river, as well as the rate of release of petroleum products from the hole were specified.

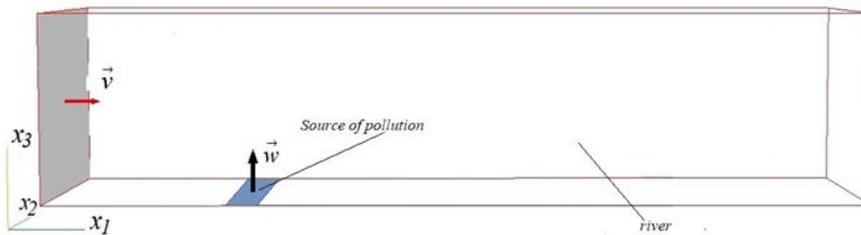


Figure 1: The calculation domain.

Table 1: Dependent variables, effective exchange coefficients and source terms in equation (1)

Conservation of	Φ	Γ_Φ	S_Φ
mass	1	0	0
x_i - momentum	u_i	$\mu + \mu_t$	$-\frac{\partial p}{\partial x_i} + \rho g_i$
Enthalpy	h	$\frac{\mu}{Pr} + \frac{\mu_t}{Pr_t}$	0
Mass of α - species	c_α	$\frac{\mu}{Sc} + \frac{\mu_t}{Sc_t}$	0
Turbulent kinetic energy	k	$\mu + \frac{\mu_t}{\sigma_k}$	$\rho(P_k + W_k - \varepsilon)$
Dissipation rate of turbulent kinetic energy	ε	$\mu + \frac{\mu_t}{\sigma_\varepsilon}$	$\rho \frac{\varepsilon}{k} (C_{\varepsilon 1} P_k - C_{\varepsilon 2} \varepsilon + C_{\varepsilon 3} W_k - R_{RNG})$

The density of oil is $\rho = 800 \text{ kg/m}^3$. For transportation of oil, it is usually heated in specially designated points. In the first case, the temperature of the oil at the time of its flow is 5°C , in the second – 15°C . The flow rate of oil from the pipeline is 0.6 m/s . The water temperature is 10°C and the flow velocity of the river: in the first case - 0.5 m/s , and in the second - 1 m/s . Mathematically, this problem is reduced to solving the Reynolds equations for turbulent flow, as well as the equations of energy and concentration of the pollutant. According to Patankar (1981), the generalized differential equation for the desired dependent variables (components of the velocity vector, components of concentrations, energy, etc.) will have the following form:

$$\frac{\partial}{\partial t}(\rho\Phi) + \frac{\partial}{\partial x_i} \left(\rho u_i \Phi - \Gamma_\Phi \frac{\partial \Phi}{\partial x_i} \right) = S_\Phi \quad (1)$$

where t is time; x_i - spatial coordinate ($i = 1, 2, 3$); ρ is the density of the liquid; u_i is the velocity component in the x_i direction, Φ is the generalized dependent variable, Γ_Φ is the mass transfer coefficient, and S_Φ is the

source term. Specific values of the listed quantities are given in Table 1. The density of the liquid is calculated from the equation of state for mixtures of liquid: p - pressure, T - absolute temperature, R - universal gas constant; c_α - mass fraction of α ; α - component of the mixture ($\alpha = 1$ - oil, 2 - water). Here k is the turbulent kinetic energy; ε is the dissipation rate of turbulent kinetic energy; μ and μ_t are the dynamic molecular and turbulent viscosities; Pr , Sc , Pr_t and Sc_t are the molecular and turbulent Prandtl and Schmidt numbers; \bar{C}_k , \bar{C}_ε , C_μ , $C_{\varepsilon 1}$, $C_{\varepsilon 2}$, $C_{\varepsilon 3}$ are the empirical constants of turbulent model; g_i is the gravity acceleration component ($\vec{g} = (0, 0, -g)$); u_i is the velocity vector component; P_k , W_k are the turbulent production terms; R_{RNG} is an additional term proposed in the RNG k- ε model.

$$p = \rho RT \sum_{\alpha=1}^2 \frac{c_\alpha}{M_\alpha} \quad (2)$$

3. Results Calculation and Analysis

The boundary-value problem is solved numerically using finite volume method (Patankar, 1981) and software PHOENICS. As a result of the numerical solution of the problem posed, spatial distributions of the fields of velocity, temperature, and concentration of petroleum products are obtained. In Figure 2 shows the concentration distribution of pollution in the river at a flow rate of 0.5 m/s in the section of the vertical plane x_1Ox_3 at different times after the start of oil release from the hole in the pipeline. The highest concentration values are located directly next to the place of oil outflow from the pipe. As the pollutant is transferred by the flow of an oil spill, its concentration decreases. Figure 3 shows the distribution of petroleum products on the water surface in the x_1Ox_2 plane. As the stain spreads in the water, the oil stain on the surface takes the form of a horseshoe, due to the fact that the flow of the pollutant affects the flow of the river

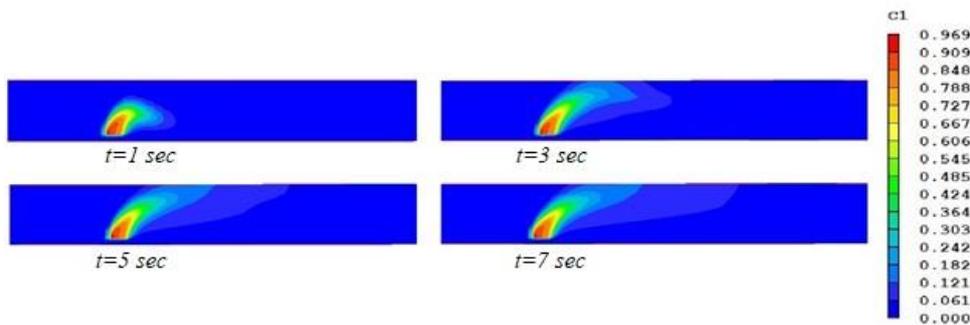


Figure 2: C_1 distribution at different times (in the x_1x_3 plane)

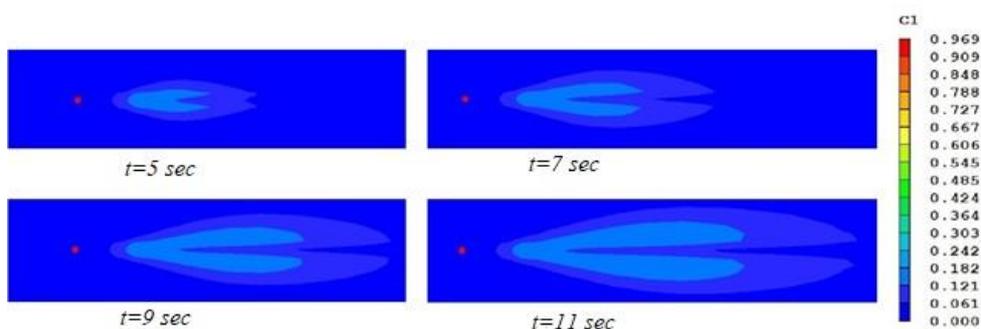


Figure 3: The distribution of c_1 on the surface of the water at different moments of time

Figure 4 clearly shows the deceleration of the current velocity near the place of emission of pollutants, both to the left and to the right of the source. In this case, with a river depth of 2 m and a river flow velocity of 0.5 m / s, oil appears on the surface of the reservoir 5 seconds after the start of the emission source. The distance from the lifting point to the oil pipeline is 3 meters. When the speed of the river flow doubles, the oil slick released during the emergency release also appears on the water surface at a greater distance from the source of oil release from the pipeline. Figure 5 shows the calculation results for a river flow rate of 1 m/s.

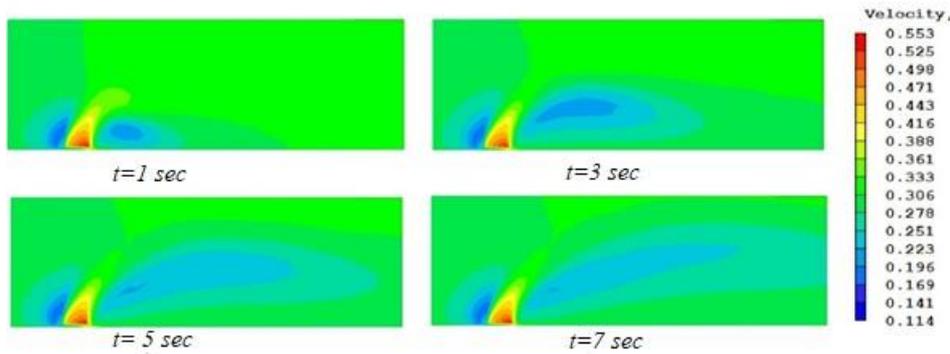


Figure 4: The distribution of the velocity field at different moments of time

Oil emulsions will be carried by the current of the river to a more distant distance and reach the surface in a greater amount of time. According to calculations, it will be possible to see the spots carried away on the surface of the river after 12 m from the place where the pipeline is located.

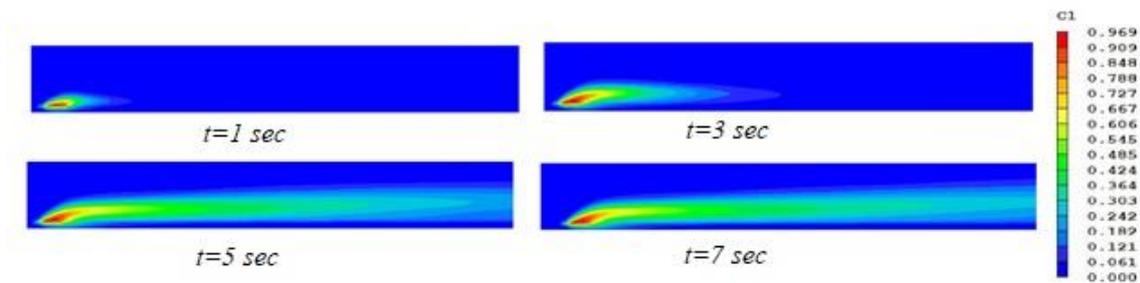


Figure 5: The distribution of the concentration of C_1 at a flow rate of 1 m/s

The volume of the leaked oil and its negative impact on the environment can be very significant even with a small hole, if it has been a long time since the fistula was formed. Nevertheless, with an increase in the size of the hole that appeared when the oil pipeline is damaged, the volume of oil release into the aquatic environment, of course, increases. Figures 6 a) and b) show the dynamics of the release of oil products for various sizes of holes in oil pipelines: a) $S_1 = 0.0225\text{ m}^2$ and b) $S_2 = 0.09\text{ m}^2$. As can be seen in Figure 6, already 3 seconds after the start of the blowout near the source of the outflow, the amount of oil flowing out increased, as evidenced by the distribution of its concentration.

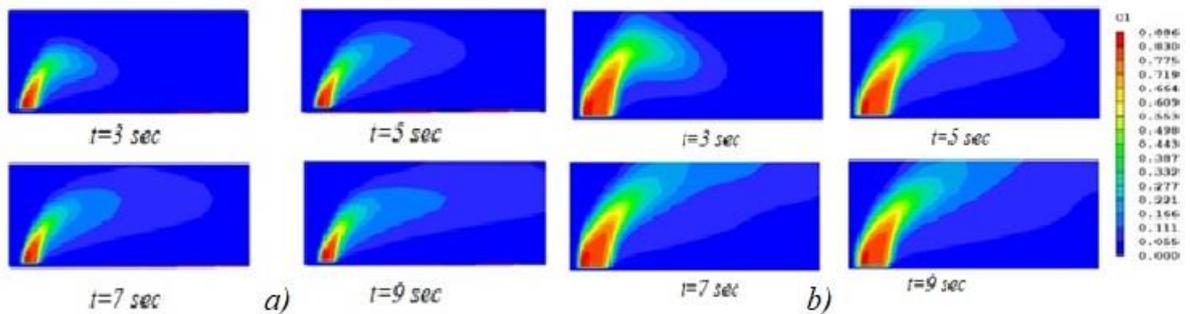


Figure 6: The distribution of the concentration of c_1 by increasing the opening of the fistula

This model makes it possible to track the spatial distribution of oil in the river bed over time for various conditions associated with both the nature of the accident and the properties and parameters of the river (flow velocity, configuration, depth, water temperature, etc.). For example, Figures 7 and 8 show the spatial distributions of the concentration of oil products in the riverbed at different points in time.

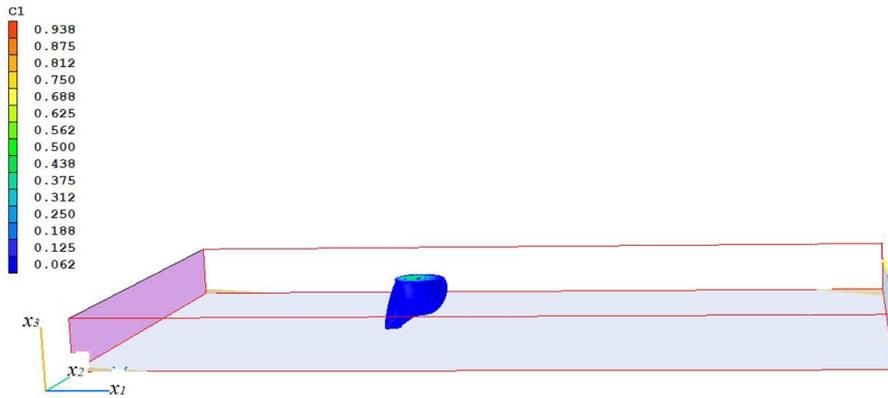


Figure 7: The distribution of c_1 at the moment $t=8$ sec.

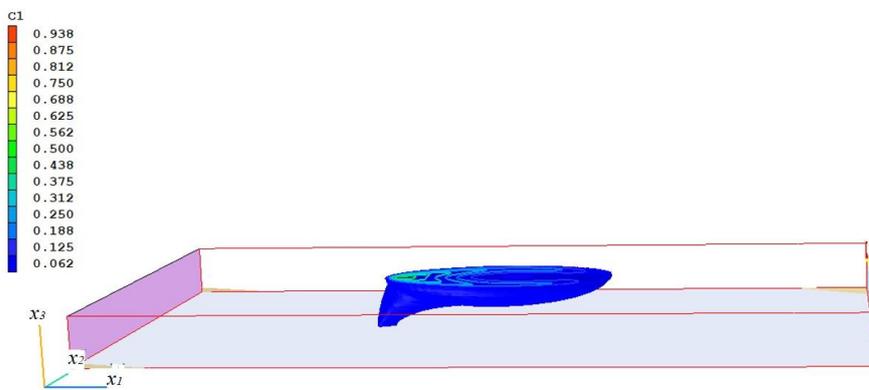


Figure 8: The distribution of c_1 at the moment $t=18$ sec.

The model developed in this paper allows one to obtain the spatial distributions of the sought functions, in particular, the velocity fields and pollutants. Thus, Figures 9 and 10 show the distribution of contamination spread at different points in time and the vector velocity field in the river bed at the corresponding time points of 8 and 18 seconds on the surface of the river.

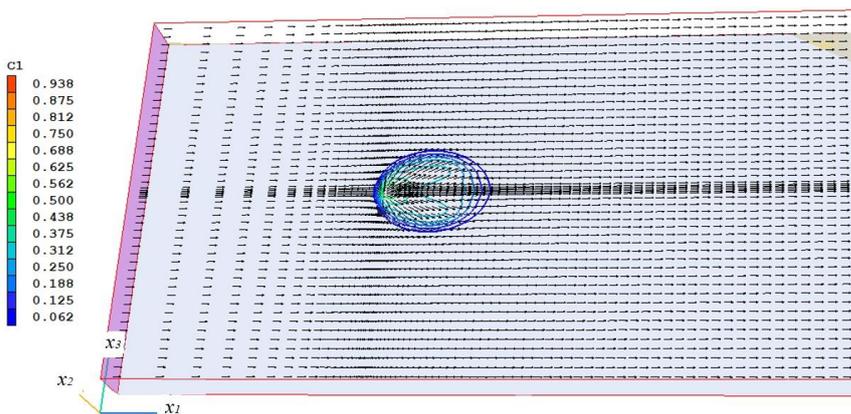


Figure 9: The distribution of c_1 and vector field of velocity at the moment $t=8$ sec.

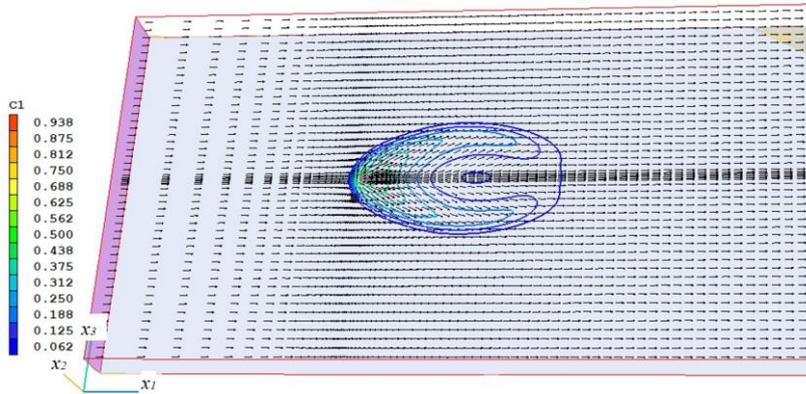


Figure 10: The distribution of c_1 and vector field of velocity at the moment $t=8$ sec.

4. Conclusion

Thus, this paper presents a mathematical model of the distribution of petroleum products in the aquatic environment when the pipeline ruptures. Using the PHOENICS software, spatial distributions of pollution in the river bed were obtained depending on various parameters. The influence of the speed of the river flow, its depth, as well as the size of the pollution source has been studied. The developed mathematical model and the created program make it possible to obtain the spatial distribution of pollutants in the river bed. The results of calculations using this model can be used to determine the location of sensors for recording pollution of the aquatic environment, as well as the dynamics of the spread of oil products over time in the river bed. This will ensure the effectiveness of the early monitoring system.

Nomenclature

$C_{\epsilon 1}, C_{\epsilon 2}, C_{\epsilon 3}$ - empirical constants	S_1, S_2 - hole area, m^2
c_α - mass concentrations of α - component	t - time, s
g - acceleration of gravity, m/s^2	T - temperature, K
k - turbulent kinetic energy, J	u_i - velocity component in x_i direction, m/s
M_α - molecular weight of α - component	W_k - the turbulent production term
p - pressure, n/m^2	x_i - cartesian coordinates, m
P_k - the turbulent production term	Γ_ϕ - coefficient of turbulent transfer
Pr - Prandtl number	ϵ - dissipation turbulent kinetic energy, m^2/s^2
R - gas constant, $J/(mol \cdot K)$	μ_t - coefficient of turbulent viscosity, $kg/(m \cdot s)$
R_{RNG} - additional term in the RNG k - ϵ model	ρ - density of the liquid, kg/m^3
Sc - Schmidt number	$\sigma_k, \sigma_\epsilon$ - empirical constants -

References

- Buckmaster, J., 1973, Viscous-gravity spreading of an oil slick, *Journal of Fluid Mechanics*, 59, 3, 481-491.
- Fanelop T. K, Waldman, G. D., 1971, Dynamics of Oil Slicks, *AIAA Journal*, 10, 4, 506-510.
- Guandalini R., Agate G., Moia F., 2017, Numerical 3D modelling of oil dispersion in the sea due to different accident scenarios, *Energy Procedia*, 125, 161-169.
- Issakhov A., Alimbek A., Zhandaulet Y., 2021, The assessment of water pollution by chemical reaction products from the activities of industrial facilities: Numerical study, *Journal of Cleaner Production*, 282, 125239.
- Juan Xu, Zongrui Hao, Yue Wang, Jin Liu, Gang Liu and Yuanming Zhang, 2019, Numerical simulation on oil spill at different positions on the back surface of a blunt body, *World Symposium on Smart Materials and Applications (WSSMA 2019)*, IOP Conf. Series: Materials Science and Engineering 649, 012027.
- Li X., Chen G., Zhu H., 2007, Modelling and assessment of accidental oil release from damaged subsea pipelines, *Marine Pollution Bulletin*, 123, 133-141.
- Liang Sun, 2012, Mathematical modeling of the flow in a pipeline with a leak, *Mathematics and Computers in Simulation*, 82, 11, 2253-2267.
- Patankar, S., 1981, Numerical heat transfer and fluid flow. Hemisphere Publishing, New York, USA. PHOENICS On-Line Information System: <www.cham.co.uk/index.php> accessed 21.06.2021. ISBN 089116-522-3