

## Optimal Design of Water Distribution Network using Two Different Metaheuristics

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Water is becoming an essential commodity for human life and is one of the most important natural resources. Public water utilities provide more than 90 percent of the world's water supply today, so a safe water distribution system is critical for any city. The importance, huge capital cost of the system, and growing city size lead to water distribution network optimization. In this work, we propose and compare two algorithms to optimize the water network design of a new neighborhood of our city, where a public cooperative is in charge of this utility. Consequently, two metaheuristic algorithms based on Tabu Search and Simulated Annealing (SOTS and HSA, respectively) arise to minimize the investment cost of a water distribution network. The experimentation suggests that both algorithms optimize the investment cost, with results that are comparable.

### 1. Introduction

More than 90 % of the in the world's water supply today is provided by public water services. Therefore, a safe drinking water distribution system is a critical element of any city. Consequently, the water distribution network design and optimization gain prime importance to minimize the cost and simultaneously maximize the network reliability and benefits. One of the design variables may be the pipe diameters, and the constraints, which are implicit functions of the decision variables, require solving the conservation of mass and energy to determine the network's pressure heads. The solution concerning the layout, design, and operation of the network of pipes should result from efficient planning and management procedures. This problem is known as Water Distribution Network Design (WDND) and requires handling a large number of variables and constraints, and in consequence, is classified as NP-hard (Yates et al. 1984).

Currently, the WDND formulations include the extension to a multi-period setting, i.e., time-varying demand patterns, which are more realistic and complex problem formulations. Some works expressed the design problem as a multi-objective optimization problem and applied a multi-objective evolutionary algorithm (Farmani et al. 2005). A genetic algorithm was developed to solve six small networks (Gupta et al. 1999), considering the velocity constraint on the water flowing through the pipes. In (Bragally et al. 2012) also regarded this constraint, but the authors used mathematical programming on bigger. Other metaheuristics were used to tackle more complex WDND formulations (Uma 2016, Mansouri & Mohamadizadeh 2017, Sedki and Ouazar 2012). An Iterative Local Search (ILS) (Decorte & Sörensen 2016) considered that every demand node has a 24 hrs water demand pattern and included a new constraint related to the limit on the maximal velocity of water through the pipes.

The growth of city sizes also requires this design process, as is the case of General Pico (La Pampa, Argentina), which needs to optimize an independent WDND of a new neighborhood of five km<sup>2</sup>, subject to multi-period demands, hydraulic restrictions, among others. A public cooperative is in charge of distributing this essential element in this city, which requires an optimization system to determine the best engineering solution in meeting established design criteria and minimizing capital costs. Our research objective is to develop an optimal solution

based on state-of-the-art optimization algorithms, which intends to support the decision-making to design, plan, and manage complex water systems.

We propose and compare Tabu Search (TS) and Simulated Annealing (SA) based metaheuristics to solve the WDND problem, optimizing the pipe diameters while at the same time minimizing the total investment cost. The TS-based method, called SOTS (Strategic Oscillations Tabu Search) adapts the constructive-destructive technique called Strategic Oscillation proposed by (Carnero et al. 2013) to enhance the search intensification and diversification capabilities. A hybrid optimization algorithm based on simulated Annealing (HSA) (Bermudez et al. 2019, 2021) enhanced with a local search procedure based on GRASP (Decorte & Sorensen 2016) is used to minimize the water network cost. We test the performance of SOTS and HSA with a real-world medium size distribution network. The main contributions of this work are the following: *i*) The adaptation of an ad hoc SO technique to the WDND problem (constructive and destructive phases of SOTS), and *ii*) Resolution of a real-world distribution network from our community.

The remainder of this article is summarized as follows. Section 2 introduces the problem definition and describes the real-world distribution network. Section 3 explains our SOTS proposal, whereas Section 4 gives the insides of the HSA to solve the WDND optimization problem. Section 5 describes the experimental design, the methodology used, and the result analysis of our proposals. The final section summarizes our conclusions and sketches out our future work.

## 2. Multi-Period Water Distribution Network Design

The optimal design of Water Distribution Networks (WDN) consists in finding the least-cost pipe configuration for a given WDN topology (node placement and network connectivity). The design of WDNs gives rise to an optimization problem called the WDN design optimization problem. The aim is to find the least cost design, in terms of optimal pipe types, that satisfies hydraulic laws and customer requirements. Consequently, the decision variables are the diameters for each pipe in the network. In this work, the WDN design problem is defined as simple-objective, multi-period, and gravity-fed one. Two restrictions are considered: the limit of water speed in each pipe and the demand pattern that varies in time. The network can be modelled by a connected graph described by a set  $V$  of  $k$  nodes,  $V = \{v_1, v_2, \dots, v_k\}$ , and a set of edges or pipes,  $L = \{l_1, l_2, \dots, l_n\}$  where  $l_j$  represents the length of the  $j$ -th pipe. If the commercially available pipe types have diameters  $d_i$ ,  $i=1:m$  with costs  $IC_i$ , then the WDN optimal design, whose objective function is the Total Investment Cost (TIC) that can be formulated as follows:

$$\min_{\mathbf{x}} TIC = \sum_i^m \sum_{j=1}^n l_{ij} IC_i x_{ij} \quad (1)$$

Where  $\mathbf{x} \in \{0,1\}^{m \times n}$  and  $x_{ij}=1$  if the  $i$ -th diameter is assigned to the  $j$ -th pipe. The objective function is constrained by: physical laws of mass and energy, conservation, minimum pressure demand in the nodes, and the maximum speed in the pipes, for each time  $\tau \in T$  (see Bermudez et al. 2021 for more details).

### 2.1 A Real Water Distribution Network

The principal motivation of this research is to get involved in solving community problems, in particular the water distribution network design. To provide some context, the water access problem in the province of La Pampa (central zone of Argentina) is a priority treatment for being a scarce natural resource. The supplier of this essential service in General Pico has to design an independent drinking WDN for a new neighborhood of five km<sup>2</sup>, minimizing the network cost through the proper selection of the pipe dimensions according to consumption and the physical laws of this type of problem. The WDN should cover an area identified as Zone 2 or Z2. Initially, the network has an extension of 1.65 km<sup>2</sup> foreseeing for the next ten years an adjacent extension of 3.4 km<sup>2</sup>, identified as the zones Z1, Z3, and Z4. The network designed for Z2 is independent but requires taking into consideration the demand of the other zones to become an extra supply network or to receive water from them (bypass).

## 3. SOTS to solve the WDND problem

Tabu Search (Glover 1989) is a widespread metaheuristic approach used to solve optimization problems. The TS algorithm is hybridized with strategic oscillations, consisting of a sequence of two destructive and one constructive phase, to increase the search intensification and diversification capabilities, giving rise to SOTS (Strategic Oscillation Tabu Search).

Concerning the problem to be solved, the definition of the solution representation and a fitness function are required. Consequently, a solution to Eq. 1 is represented as an integer  $m$ -vector  $\mathbf{s} = \{s_1, s_2, \dots, s_m\}$ , where  $m$  is the number of pipes present in the network. The  $i$ -th component,  $s_i$ , can take  $n$  different integer values, belonging to the set  $\{d_1, d_2, \dots, d_n\}$  of commercially available pipe diameters for the network. The following fitness function is applied where,  $g(\mathbf{s})$  takes into account constraint violations:

$$H = \begin{cases} TIC & \text{if } \mathbf{s} \text{ is feasible} \\ TIC + g(\mathbf{s}) & \text{if } \mathbf{s} \text{ is infeasible} \end{cases} \quad (2)$$

The solution search space size is of  $m^n$  order, if there are no additional restrictions on the pipe diameters. In real design problems with a large number of pipes in the network, it is necessary to have algorithms that allow an automatic and optimal selection of diameters for each of them. Given a feasible solution, the destructive phase drives the search to cross the feasibility boundary. Then move rules are modified and the constructive phase starts. The search returns toward the feasible region until a condition is satisfied. The use of standard TS mechanisms avoids going back over previous search trajectories. For the solution of the WDND represented by Eq.1, the local search procedure based on SOTS is described as follows.

Let  $\mathbf{s}$  be a candidate solution, a neighborhood  $N_1(\mathbf{s})$  is built during the destructive phases. For the first destructive one, the neighborhood is the set of solutions obtained by decrementing from  $\mathbf{s}$  one diameter if the move is allowable. This condition is verified when the element of the Recency-based Tabu Matrix associated with the diameter and the pipe is zero. That matrix has an  $m \times n$  dimension, a non-zero element indicates that the move is forbidden because it was done to obtain a recent solution. Furthermore, its value is the number of remaining iterations until the Tabu Tenure Period,  $pt$ , for this move is elapsed. The second considered destructive phase uses the problem knowledge provided by the simulation tool. The destruction moves, as defined above, are performed in decreasing order for those pipes that most loosely verified the velocity and pressure constraints. During both destructive phases, the search crosses the feasibility boundary. Then the constructive phase begins, and the neighborhood  $N_3(\mathbf{s})$  is defined as the set of solutions obtained increasing by one the  $i$ -th pipe diameter if the move is not tabu. The constructive phase ends after  $r$  iterations into the feasible region.

The search information stored in this matrix is used only in the constructive phase. The evaluation function  $H$  corresponding to the  $i$ -th allowable increasing diameter for the  $j$ -th pipe is penalized in proportion  $\alpha$  to the value of the  $[i,j]$  element of this matrix, in order to direct the search to less frequently visited regions. After  $ph$  iterations, the frequency-based tabu list is reset.

The candidate solution neighborhood cardinality is of order  $m$ , and each element of this neighborhood is evaluated *maxiter* number of times. Therefore, each run of SOTS consumes a (*maxiter*\* $m$ ) number of function evaluations. The computational time associated with that procedure can be excessive for large networks. Therefore, raw fitness (*Rfi*) is computed, and solutions belonging to the neighborhood are sorted according to *Rfi*. Then a function evaluation call, which includes a simulation step, is performed. The procedure stops when a feasible non-tabu solution is reached, otherwise the first sorted solution is selected. Also, the best-found solution ( $\mathbf{s}^*$ ) is saved. Aspiration criteria are applied to determine when the Tabu matrix and the frequency-based tabu table can be overridden to avoid missing good solutions. Algorithm 1 shows the general pseudocode of SOTS.

#### 4. HSA to solve the WDND problem

The Simulated Annealing (SA) (Kirkpatrick et al. 1983) is an optimization method, which explores through the solution space using a stochastic hill-climbing process. The more efficient SA formulations are based on two cycles: one external for temperatures and other internal, named Metropolis. The same Markov chain length in the Metropolis cycle is usually used for each temperature  $T$  (a control parameter with  $T > 0$ ). The SA algorithm can be seen like a sequence of Markov chains, where each Markov chain is constructed for descending values of  $T$ . The HSA solver proposed to optimize the WDND problem in (Bermudez et al. 2021) is an algorithm based on SA, taking its advantages and adapting to the problem as the Algorithm 2 shows.

HSA begins with the initialization of the temperature (line 2). The choice of the right initial temperature plays a crucial role in the HSA performance to find good solutions, this procedure is explained in (Bermudez et al. 2021, Hernández et al. 2019). Once the initialization process ends, an iterative process starts (lines 5 to 20). The first step in the iteration involves a hybridization to intensify the exploration into the current search space. In this way, a feasible iteration involves a hybridization to intensify the exploration into the current search space. In this way, a feasible solution,  $\mathbf{s}_1$ , is obtained by applying the MP-GRASP local search (De Corte and Sorensen 2016) to  $\mathbf{s}_0$  (line 8), and then a greedy selection mechanism is performed (lines10-12). Therefore,  $\mathbf{s}_0$  can be replaced by  $\mathbf{s}_1$  if it is better than  $\mathbf{s}_0$ . In the next step, a perturbation operator is used to obtain a feasible neighbor,  $\mathbf{s}_2$ , from  $\mathbf{s}_0$  (line 13), to explore other areas of the search space.

This perturbation randomly changes some pipe diameters. If  $s_2$  is worse than  $s_0$ ,  $s_2$  can be accepted under the Boltzmann probability (line 15, second condition). In this way, the search space exploration is strengthened when the temperature ( $T$ ) is high. In contrast, at low temperatures the algorithm only exploits a promising region of the solution space, intensifying the search. To update  $T$ , a random cooling schedule (Bermudez et al. 2018) is used (line 19), which combines three traditional cooling schemes (the proportional Kirkpatrick et al. 1983], exponential Kirkpatrick et al. 1983], and logarithmic Hajek 1988] schemes) in only one schedule process. The cooling schedule is applied after a certain number of iterations ( $k$ ) given by the Markov Chain Length (MCL) (line 18). Finally, HSA ends the search when the total evaluation number is reached or  $T = 0$ .

```

1. Given: initial solution  $s_0$ 
2. if  $s_0$  is feasible
3.   Set Phase=1 /* destructive phase */
4. else
5.   Set Phase=0 /* constructive phase */
6. end
7. Set  $s^*=s=s_0$ 
8. for  $i=1$  to #maxiter
9.   if Phase =1
10.    if  $i \bmod 2=1$  or there is no feasible solution yet.
11.      Generate neighborhood  $N_2(s)$  using specific problem knowledge
12.    else
13.      Generate neighborhood  $N_1(s)$  by blind decrementing one diameter at a time
14.    end
15.    Evaluate neighborhood
16.    Get best neighbor  $s'$ 
17.    if condition1=true
18.      Phase = 0
19.    end
20.  else
21.    Generate neighborhood  $N_2(s)$  and evaluate
22.    Get best neighbor  $s'$ 
23.    if condition2=true
24.      Phase=1
25.    end
26.  end
27.   $s=s'$ 
28.  Update Recency and Frequency Tabu Matrices
29.  if  $s \prec s^*$ 
30.     $s^*=s$ 
31.  end
32. end
33. return  $s^*$ 

```

Algorithm 1: SOTS algorithm to solve the WDND optimization problem.

## 5. Experimental Design and Result Analysis

This section describes the experimental design carried out to analyze the behavior of the SOTs and HSA to solve the WDND problem. The GP-Z2-2020 network is composed of 222 domestic demand nodes and only one water reservoir. For this case study, the available pipes are defined as a set of diameters  $D=\{63, 90, 110, 125, 315, 400, 450, 630\}$ , each associated with a cost given for  $\text{cost}=\{2.85, 5.90, 8.79, 11.00, 69.10, 110.89, 140.15, 273.28\}$ . All roughness is 110 mm. The area is residential with demand according to the current distribution of the customers in 584 plots but considering a development pattern.

The daily pattern demand corresponds to the summer period (based on the model demand of historical records) having a maximum resolution of one hour. The total number of possible combinations of design for a set of 8 commercial pipe types and 282 pipes is 8282 that makes the instance in a difficult case to solve; this shows the importance of optimization.

```

1.  $k = 0$ 
2.  $initTemp(T)$ 
3.  $initialize(s_0)$ 
4.  $TIC_0 = evaluate(s_0)$ 
5. repeat
6.   repeat
7.      $k = k + 1$ 
8.      $s_1 = MP-GRASP_{LS}(s_0)$ 
9.      $TIC_1 = evaluate(s_1)$ 
10.    if  $TIC_1 < TIC_0$ 
11.       $s_0 = s_1$ 
12.       $TIC_0 = TIC_1$ 
13.    end
14.     $s_2 = perturbation\_operator(s_0)$ 
15.     $TIC_2 = evaluate(s_2)$ 
16.    if  $(TIC_2 < TIC_0)$  or  $(exp^{-\{(TIC_2 - TIC_0)/T\}} > random(0,1))$ 
17.       $s_0 = s_2$ 
18.       $TIC_0 = TIC_2$ 
19.    end
20.  until  $(k \bmod MCL == 0)$ 
21.   $update(T)$ 
22. until stop criterion is met
23. return  $s_0$ 

```

Algorithm 2: HSA to solve the WDND Optimization Problem

We present the SOTS and HSA parametric setting to solve the real-world WDND problem. The SOTS parameters used are:  $pt=10$ ,  $ph=20$ ,  $r=4$ ,  $\alpha=0.15$  and  $maxiter=1700$ . HSA employs the random cooling scheme (Bermudez et al. 2018) and a seed temperature set in 100 (see Bermudez et al. 2019 for a justification of this parameter selection). Furthermore, these both proposals use the EPANET 2.0 toolkit [Rossman 1999] to solve the hydraulic equations. The WDND solution representation and its operators are in (Bermudez et al. 2021).

Table 2. Comparison of network pipe layouts obtained by HSA and SOTS

diameter mm	# pipes HSA	# pipes TS-OS
63	209	198
90	8	22
110	10	22
125	14	31
315	40	8
400	1	3
450	2	0
#pipes	284	284
Cost	347596	365477

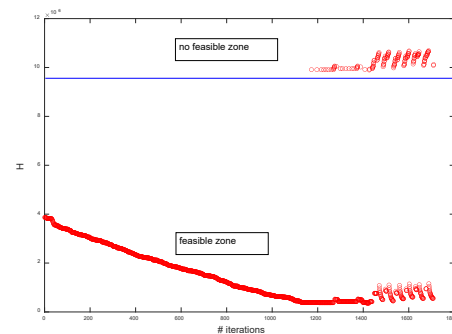


Figure 2. Solution path followed by the SOTS in terms of  $H$

Following with the result analysis, Table 2 shows the values and details of the best solutions found so far by the two proposed methodologies. As can be seen, there is a difference of 5.14% in the total investment cost between the two methodologies. In no case is the larger diameter pipe (630 mm) used in the design of the network. Figure 2 shows the evolution of the SOTS from a solution with a  $TIC$  of  $4 \times 10^6$  order. The lowest value obtained corresponds to a  $TIC = 365477$ . A first zone can be identified where the intelligent destructive phase predominates and where a steady decrease of the evaluation function is observed. At a certain point, the destructive phases start to cross the infeasible zone. Unfeasible solutions are penalised and shown with higher values of  $H$ . In this zone, the alternation of phases allows oscillation until better  $TIC$  values are obtained. If no stopping criterion is applied, a zone is reached where the algorithm no longer produces better solutions and oscillation alternates between penalised and suboptimal values.

## 6. Conclusions

This paper presents two metaheuristic approaches to solving a real WDND problem. On the one hand, we implement a TS algorithm enhanced with an ad hoc SO technique consisting of a sequence of two destructive phases and one constructive phase to improve the intensification and diversification in the search, resulting in a SOTS algorithm. On the other hand, we present a hybrid algorithm based on SA methods, called HSA combined with the GRASP, as a local search.

The performance of these two proposals is analysed by considering the solution quality. SOTS algorithm can start the search from solutions with very high total investment costs and reaches good solutions in the search's early stages, with relatively low computational effort compared to the HSA. This is mainly due to the implemented intelligent destruction phase. However, as the search progresses, the decrease in the total investment costs found becomes smaller, the exploratory capacity decreases, and the number of times the evaluation function is executed increases. Similar behaviour can be observed in HSA. The tools available for avoiding local minima are different for the two methods: short-term memory and long-term memory in the case of SOTS, and the acceptance of worse solutions when the temperature is high in the case of HSA. Considering that the space of possible solutions for the given problem is of the order of 10256, the possibility of exploring different regions of the problem using a single search technique is limited, and efforts to design solvers for the WDND should consider cooperative strategies between the two methods. Future work aims to develop hybridized algorithms, combining the advantages of SOTS and HSA and avoiding its weaknesses.

## References

- Bermudez C., Salto C., Minetti G., 2019, Solving the multi-period water distribution network design problem with a hybrid simulated annealing, *Computer Science – CACIC 2018*. Springer International Publishing, 1–10.
- Bermudez C., Alfonso H., Minetti G., Salto C., 2020. Hybrid simulated annealing to optimise the water distribution network design: A real case. *Computer Science – CACIC 2020*, Springer International Publishing, 19–34.
- Bragalli C., D'Ambrosio C., Lee J., Lodi A., Toth P., 2012, On the optimal design of water distribution networks: a practical MINLP approach, *Optimization and Engineering*, 13(2), 219–246.
- Carnero M., Hernández J., Sánchez M., 2013, A new metaheuristic based approach for the design of sensor networks, *Computers Chemical Engineering*, 55, 83–96.
- De Corte, Sörensen K., 2016, An iterated local search algorithm for water distribution network design optimization, *Network*, 67(3), 187–198.
- Hajek B., 1988, Cooling schedules for optimal annealing. *Mathematics of Operations Research*, 13(2): 311–329.
- Hernandez J., Salto C., Minetti G., Carnero M., Sanchez M.C., 2019, Hybrid Simulated Annealing for Optimal Cost Instrumentation in Chemical Plants, *Chemical Engineering Transactions*, 74, 709-714.
- Farmani R., Walters G. A., Savic D. A., 2005, Trade-off between total cost and reliability for anytown water distribution network, *Journal of Water Resources Planning and Management*, 131(3), 161–171.
- Glover F., 1989, Tabu Search – Part 1. *ORSA Journal on Computing*. 1(2), 190–206.
- Gupta I., Gupta A., Khanna P., 1999, Genetic algorithm for optimization of water distribution systems, *Environmental Modelling & Software*, 14(5), pp. 437–446.
- Kirkpatrick S., Gelatt Jr C.D., Vecchi M.P., 1983, Optimization by simulated annealing. *Science*, (220):671–680.
- Mansouri R, Mohamadizadeh M., 2017, Optimal design of water distribution system using central force optimization and differential evolution, 7(3), 469-491.
- Rossman L., 1999, The EPANET Programmer's Toolkit for Analysis of Water Distribution Systems. doi: 10.1061/40430-199
- Sedki A., Ouazar D., 2012, Hybrid particle swarm optimization and differential evolution for optimal design of water distribution systems, *Adv. Eng. Informatics*, 26, 582–591.
- Uma R., 2016, Optimal design of water distribution network using differential evolution, *International Journal of Science and Research (IJSR)*, 5(11), 1515–1520.