

Mathematical Modeling of the Kinetics of Released Antioxidant Compounds During Solid-Liquid Extraction for a Nutraceutical Beverage from *Ilex Guayusa*, *Vernonanthura Patens*, and Cocoa Husk Plants

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Mechanistic model parameters are estimated to describe the kinetics of released antioxidant compounds (phenolic compounds) during the solid-liquid extraction of a potential nutraceutical beverage obtained from *Ilex guayusa*, *Vernonanthura patens*, and Cocoa Husk plants. While the concentration evolution for the solvent phase is measured by spectrophotometry, the concentration for the solid phase is determined by mass balances. The mass transfer coefficients are then estimated using the Gekko library by fitting the model parameters with the experimental data for both the solvent and solid phases. Combining experimental data with a mechanistic model for solid-liquid extraction and programming-numeric computing platforms allows for obtaining a robust model (small MSLE); that considers mass transfer phenomena and estimates the required time for the highest percentage extraction of the phenolic compounds. This work provides a potential scheme that can be used for determining predictive analytical models through estimating parameters in Gekko using the experimental kinetics for different raw materials in the food industry and incorporating them into simulators through a connection between simulators with Matlab or Python.

1. Introduction

In the last decades, the food industry has increased the development of plant-based beverages with antioxidant properties, known as nutraceutical beverages. Herbal infusions are a popular nutraceutical beverage in traditional medicine, used in tumors treatments; hypertension; and gastrointestinal diseases (Paladines-Santacruz et al., 2021). Quijano-Avilés et al., (2021) developed an herbal infusion comprised of three plants (*Ilex guayusa*, *Vernonanthura patens*, and Cocoa Husk), and they studied the correlation between the antioxidant properties of the infusion with the different proportions of the plants (Quijano-Avilés et al., 2021). This infusion has a high content of polyphenols (chlorogenic acids, flavonoids, phenolic acids) with a high antioxidant capacity, providing health benefits and the prevention of diseases associated with metabolic syndromes (Paladines-Santacruz et al., 2021).

Food companies and researchers seek to expand and strengthen their technical knowledge in developing nutraceutical products (Pettinato et al., 2021). This process development requires considering different factors, such as the properties of the raw material and operating conditions (Quijano-Avilés et al., 2021). Furthermore, industries have relied on process simulators (Casa & Miccio, 2022) and mathematical models to represent the

reality of their processes (Setford et al., 2019). The advantages of having a mathematical model with experimental data are: that it adequately describes the behavior of the process and allows forecasting the results according to certain conditions (Cerón-Montes et al., 2021). The mechanistic solid-liquid mathematical model proposed by Espinoza-Pérez et al., 2007, consisting of two simultaneous algebraic equations obtained from the analytical solution of the macroscopic mass transfer equations, can be used to evaluate the mass transfer coefficient in the liquid phase and the diffusivity coefficient in the solid phase. This model simulates the extraction of different antioxidant compounds from the following raw materials: coffee (Espinoza-Pérez et al., 2007), grapes (Setford et al., 2019), and vanilla. The mathematical model is also validated with experimental data, ensuring a high prediction by minimizing the error with least squares and adequate parameters estimation with a confidence interval of 95% (Rodríguez-Jimenes et al., 2013).

In this work, the experimental kinetics of a mixture of three plants (*Ilex guayusa*, *Vernonanthura patens*, and Cocoa Husk) with nutraceutical properties are obtained from a solid-liquid extraction. The experimental kinetics consists of measuring the concentrations of the nutraceutical compounds in the extract at different times. Then, using the model proposed by Espinoza-Pérez et al. (2007), the experimental data is adjusted through the Gekko library in Python (Beal et al., 2018), obtaining the time and the mass transfer coefficients in both; solid and liquid phases.

2. Material and Methods

2.1 Mathematical model

Solid-liquid extraction is a mass transfer operation that involves contact between two phases: the refined (solid) and the extract (solvent). This operation comprises four phenomena. The solvent penetration, into the solid medium, by the fluid flow. The dissolution of the extractable compound (solute) from the solid, by the solvent. The mass transfer by solute diffusion through the pores and the mass transfer by convection from the interface of pores to the solvent phase. A mechanical model developed by Espinoza-Pérez et al. (2007) considers these four phenomena and can be summarized in Eq(1-3). Eq(1) represents the total mass transfer from the interface to the solvent, Eq(2) shows the mass transfer from the solid to the interface, and Eq(3) represents the equilibrium between the solid-liquid phases.

$$\varepsilon \frac{dc_\gamma}{dt} = k_{c\gamma} a (c_{\gamma i} - c_\gamma), \quad (1)$$

$$(1 - \varepsilon) \frac{dc_\beta}{dt} = k_{c\beta} a (c_{\beta i} - c_\beta), \quad (2)$$

$$c_{\gamma i} = K c_{\beta i}, \quad (3)$$

where a is the surface: volume ratio of a sphere ($a = \frac{6}{D_p}$); with D_p the particle diameter, ε is the fraction between the volume of the solvent and the total volume. The mass transfer coefficients in the solvent and the solid are $k_{c\gamma}$ and $k_{c\beta}$, respectively. The concentrations of the solute in the solvent and in the solid are c_γ and c_β , respectively. The subscript i denotes the interface in the system, and K is the equilibrium constant.

Espinoza-Pérez et al. (2007) propose an analytical solution for Eq(1-3) using Laplace transforms. From this, the average concentrations for both; the solute and the solvent in the solid phase are in Eq(4) and Eq(5), respectively.

$$c_\beta = c_{\beta 0} (C_1 e^{r_1 t} + C_2 e^{r_2 t}), \quad (4)$$

$$c_\gamma = c_{\beta 0} (C_3 e^{r_1 t} + C_4 e^{r_2 t}), \quad (5)$$

where $c_{\beta 0}$ is the initial concentration of the solute in the solid, and t represents the time. The coefficients C_1 , C_2 , C_3 , C_4 , r_1 , and r_2 represent dimensionless constants in the model.

2.2 Experimental methodology

Extraction of phenolic compounds for the nutraceutical beverage

Figure 1 depicts the production process of the nutraceutical beverage, from the plants drying to the extraction of the phenolic compounds in an aqueous medium at atmospheric pressure. The feed stream (stream 4) comprises a mix of Cocoa Husk (stream 1), *Ilex guayusa* (stream 2), and *Vernonanthura patens* (stream 3) in a ratio of 1:1:1 (Espinoza-Vasquez et al., 2023). Stream 4 feeds a dryer (DE-101) to reduce the moisture content

to less than 12% at a constant temperature. The dryer is supported by a heat exchanger, which heats the air (stream 5) from ambient temperature to 45°C (stream 6). Subsequently, the dry stream (stream 8) enters a mill (stream 9) to reduce the particle size ($2.5 \times 10^{-5} - 5 \times 10^{-4}$ m) to increase the transfer area (stream 9). Stream 9 enters a sieve (S-101) to ensure suitable particle size before arriving at the extractor. Stream 10 represents the fine solids, while stream 11 represents the coarse solids that must be back in the mill. Finally, stream 10 feeds an extractor (E-101), where the heat exchanger (HX-102) preheats the water (stream 12) to a temperature of 60° and enters the extractor using a ratio of 1:10 (w/s). The extractor operates for 15 minutes to extract the phenol content from the milling of plants. The outlet stream from the digester (stream 14) is filtered in the filter (P-101), separating the residue husk (stream 16) and the beverage (stream 15) (Quijano-Avilés et al., 2021).

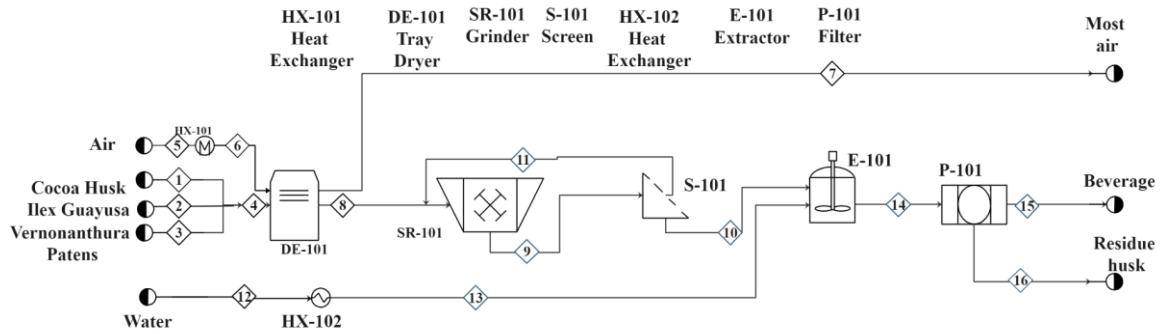


Figure 1: Production process of the nutraceutical beverage or extraction of phenol compounds

Quantification of phenolic compounds

In the extraction stage for the nutraceutical beverage, 2 ml of samples were collected every 1.5 minutes during the 10.5 minutes of operation, ensuring the concentration in the solvent is constant. Additionally, a beverage sample is stored after 15 minutes of operation to determine the total concentration of phenolic compounds. The solute concentrations in the solvent phase are determined by the Folin-Ciocalteu method because plant phenolic compounds react with antioxidant agents. This method means mixing 2uL of the extract with 80 uL of 7.5% v/v sodium carbonate solution and 100 uL of a Folin-Ciocalteu reagent solution at a 1:10 volumetric ratio (Quijano-Avilés et al., 2021). The absorbance of the samples is measured using the BioTek Cytation™ 5 instrument at a wavelength of 760×10^{-6} m, and the concentration is determined using a relation between absorbance and gallic acid concentration. Eq(6) shows the linear regression between absorbance and gallic acid concentration with a coefficient of determination of 0.991 (Patiño & Prado, 2022).

$$[c] = (5.26 * 10^5) * A - 7.90 * 10^{-3} \quad (6)$$

where c represents the concentration (kg m^{-3}), and A is the absorbance (m).

On the other hand, the solute concentration in the solid phase is determined through the mass balance of the phenolic compounds. Eq(7) represents the mass balance using concentration and volume. The solid phase volume can be calculated by the relationship between its mass (0.1 kg) and its solid density (1360 kg m^{-3}) (Patiño & Prado, 2022).

$$c_{\beta 0} V_{\beta} = c_{\beta, t=x} V_{\beta} + c_{\gamma, t=x} V_{\gamma} \quad (7)$$

where V represents the volume, and c is the concentration of the solute. The subscript t is the time, and $t = x$ indicates an extraction during the time x . The subscripts β and γ represent the phases of solid and solvent, respectively. The subscript 0 indicates the initial condition.

2.3 Parameter Estimation

In this work, the monitored extractable materials (solute) are phenolic compounds during the solid-liquid extraction in a mixture of the *Ilex guayusa*, Cocoa Husk, and *Vernonanthurra patens* plants. The estimation of parameters of the differential equations Eq(1-3) and the average concentration Eq(4-5) are fitted using the experimental data. For Eq(1-3), the parameters are the mass transfer coefficient for both the solvent ($k_{c\gamma}$) and the solid ($k_{c\beta}$) phases, as well as the equilibrium constant (K). Gekko library is a Python package developed for toptimizingdifferential-algebraic equations and mixed integers, solving problems of linear programming, quadratic programming, parameter estimation, dynamic simulation, and nonlinear predictive control (Beal et al.,

2018). For this reason, we use the Gekko library in Python to fit these parameters using the resolution mode '5 IMODE' because we require solving a set of differential equations simultaneously (Beal et al., 2018). Furthermore, how the library requires the range for the parameters to be estimated, we set these ranges in $k_{c\beta} \in [0,1]$, $k_{c\gamma} \in [0,1]$, $K \in [0,1]$. On the other hand, for Eq(4-5), the estimated fixed parameters are the constants C_1 , C_2 , C_3 , C_4 , r_1 , and r_2 . These constants of Eq(4-5) belong to the analytical solution of Eq(1-3). The resolution mode used in Gekko is '2 IMODE' because the regression is non-linear, and time is a dependent variable in the model. Finally, we calculate the mean square logarithmic error (MSLE) between the experimental data and the values predicted by the mechanistic model to determine the model precision using the Scikit-learn library (Pedregosa et al., 2011). Table 1 depicts the typical values of some model parameters determined experimentally given in Patiño & Prado (2022) and thus are set constant in the parameter estimation process.

Table 1: Parameters obtained experimentally and fixed during the fitting.

| Parameter | Value | Unit |
|---------------|--------------------|----------------------------|
| ε | 0.9941 | – |
| a | 3.79×10^4 | $\text{m}^2 \text{m}^{-3}$ |
| $c_{\beta 0}$ | 32.24 | kg m^{-3} |

3. Results

Table 2 shows the estimated values of the eight parameters defined in both sets of equations using the Gekko library. The mass coefficients are determined to establish the phase with the highest resistance between the solid-liquid extraction, indicating whether the extraction is slow due to the characteristics of the liquid (viscosity, temperature) or the plant characteristics (solubility, particle size). The magnitude of the mass transfer coefficient in the solvent phase ($2.28 \times 10^{-6} \text{ m s}^{-1}$) is greater than the corresponding coefficient in the solid phase ($1.30 \times 10^{-8} \text{ m s}^{-1}$). This difference indicates that the mass transfer between the interface and the solid phase presents more resistance in mass phenomena compared to the interface and the solvent phase.

Table 2: Estimated parameters in the differential equations and the mechanistic model.

| Equations | Constants | Value | Unit |
|-----------|---------------|------------------------|-------------------|
| (1-3) | $k_{c\gamma}$ | 2.28×10^{-6} | m s^{-1} |
| | $k_{c\beta}$ | 1.30×10^{-8} | m s^{-1} |
| | K | 6.89×10^{-2} | – |
| (4-5) | C_1 | 8.99×10^{-1} | – |
| | C_2 | 1.00×10^{-1} | – |
| | C_3 | -6.71×10^{-3} | – |
| | C_4 | 6.71×10^{-3} | – |
| | r_1 | -1.92×10^{-2} | – |
| | r_2 | -2.36×10^{-5} | – |

On the other hand, the results obtained from the six constants of the analytical solution (Eq(4-5)) are used to carry out the extraction curve of the phenolic compounds in the solid and liquid phases. Figure 2 shows the non-linear regression obtained from Eq(4-5) and the experimental data of the concentrations.

3.1 Evolution of phenol compounds during solid-liquid extraction in the solvent phase

Figure 2a shows that in the solvent, the solute concentration (phenolic compounds) increases quickly until reaching 0.212 kg m^{-3} after 250 seconds. However, the solvent phase concentration tends to be constant with values between 0.213 a 0.214 kg m^{-3} when time is beyond 250 seconds. This almost flat behavior is because the extraction process reaches the equilibrium between the liquid and solid phases. On the other hand, vertical lines in Figure 2 are the times required for obtaining an extraction percentage of phenolic compounds of 90% (dashed lines), 70% (dashed-dotted lines), and 50% (dotted lines). The minimum operating time of the extractor to obtain a percentage of 50% of the phenolic compounds from the raw material must be greater than 42 seconds, and for achieving an extraction of around 90% is necessary an operating time of 243 seconds. Finally, the MSLE error for the non-linear regression in the solvent phase (Eq (4)) is 1.5×10^{-3} . This value assures us that the predicted values are not far from the experimental data.

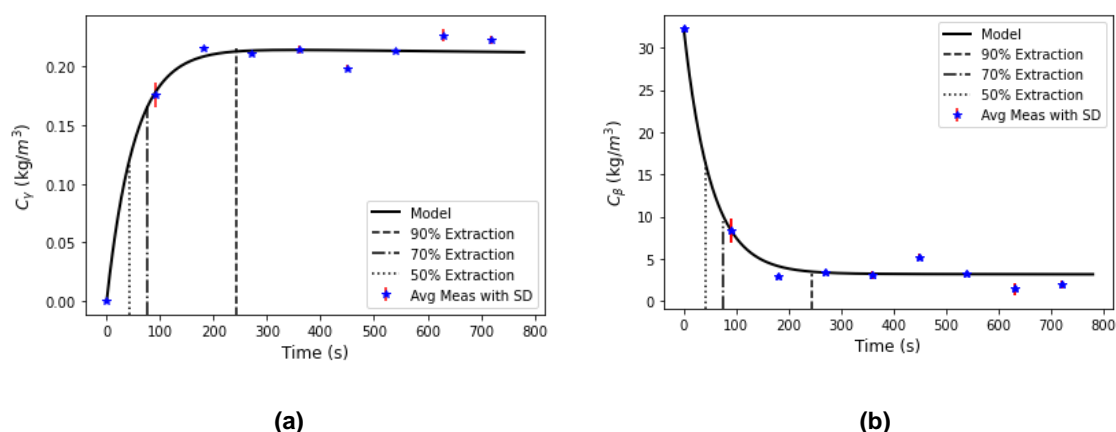


Figure 2: Experimental and simulated evolution of phenol compounds during solid-liquid extraction: (a) Solvent phase, and (b) Solid phase. The black line indicates the data obtained by the non-linear regression. "Avg Meas with SD" is the average concentrations (blue "*" symbol) with their standard deviation (red vertical bar). The vertical dotted, dashed-dotted, and dashed lines represent the extraction percentage of the phenolic compounds at 50%, 70%, and 90%, respectively.

3.2 Phenol compounds evolution during solid-liquid extraction in solid phase

Figure 2b shows that the solute concentration (phenolic compounds) in the solid phase decreases quickly until reaching 3.45 kg m^{-3} after 250 seconds. However, the solid phase concentration tends to be constant between 3.45 and 3.15 kg m^{-3} . When the operating time of the extractor is 243 seconds, 10% of the phenolic compounds are not transferred from the solid phase (plants) to solvent due to the equilibrium in the system. For example, the extraction percentages obtained at 243 seconds (90.06%) and 800 seconds (90.15%) are almost similar and vary around 0.07%. Finally, the MSLE error for the non-linear regression in the solid phase (Eq (5)) is 1.5×10^{-1} , also giving the certitude that estimates values are close to the experimental.

4. Conclusions

A non-differential mechanistic model allows for representing the extraction kinetics for the case of solid-liquid extraction. In this work, the parameters of the mechanical model for the phenolic compounds extracted from a mixture of three plants (*Ilex guayusa*, Cocoa Husk, and *Vernonanthura patens*) are estimated and validated with experimental kinetics. The values of the mass transfer coefficients of the solid and the solvent are determined by a dynamic estimation using the Gekko library, obtaining values of $1.30 \times 10^{-8} \text{ m s}^{-1}$, and $2.28 \times 10^{-6} \text{ m s}^{-1}$, respectively. Combining experimental data with a mechanistic model for solid-liquid extraction and programming and numeric computing platforms allows obtaining a robust model (small MSLE) that considers mass transfer phenomena. The model for extracting phenol compounds will enable us to determine the necessary time to reach a concentration value or the maximum extraction percentage by the user, considering the balance between the phases. Herein, the highest extraction percentage of phenolic compounds is 90% in 243 seconds, and this value is subject to the experimental data of the operating conditions described above. Potentially, this work can be adapted to determine predictive analytical models by estimating parameters in Gekko with experimental kinetics for different raw materials, and incorporating them into simulators, using the connection between the Simulator with Python or Matlab.

Nomenclature

A – absorbance, m

a – specific surface for mass transfer, $\text{m}^2 \text{m}^{-3}$

c – phenolic compounds concentration, kg m^{-3}

$C_{1,2,3,4}$ – set of constants required for Eq(4-5), -

D_p – particle diameter, m

k_c – global mass transfer, m s^{-1}

K – distribution constant, -

$r_{1,2}$ – set of constants required for Eq(4-5), -

t – time, s

V – volume, m^{-3}

Greeks

ε – volume fraction of solvent, -

Subscripts

0 – at initial or a reference

i – at the interface

x – indicates at a specific time

β – for disperse phase (solid)

γ – for continuous phase (solvent)

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