



Incorporation of Epistemic Uncertainties in Resource Conservation Networks with Multiple Resources

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Pinch Analysis is an effective technique to optimise Resource Conservation Networks (RCNs). These networks are widely used in process industries like petrochemical, pharmaceutical, and cement. Due to climate change, regulatory compliances, and market competition, process industries operate in epistemic conditions that impact their production efficiency and end-product quality. Process industries also use multiple resources to improve economic efficiency. Deployed RCNs must be designed to incorporate epistemic uncertainty while using multiple resources. Existing literature indicates that optimisation of RCNs using a single resource with epistemic parameters has been done. This work addresses the targeting of multiple resources present in RCNs, under an epistemic environment. Prioritized Cost formulation is extended to accommodate epistemic uncertainty by representing uncertain parameters using interval numbers. The proposed methodology identifies the parameter values favourable to utilising multiple resources. This methodology is illustrated through a case study on the solvent utilization network. The results from the case study demonstrate that adding another resource, under best-case condition, reduces total cost by 37.1 % from \$ 40,580 to \$ 25,516. Under worst-case condition, another resource is not required. An analysis of Prioritized Cost variation with different possible combinations of parameter values is presented using this case study.

1. Introduction

Effective and efficient utilisation of resources is critical for industrial operations in various sectors like manufacturing, agriculture, energy, and transportation. Optimal utilisation is achieved by designing RCNs to minimise the total cost of resources, offering market competitiveness to industries. RCNs are optimised using Pinch Analysis (PA), a mathematical approach based on resource allocation such that existing sources are utilised to the fullest to fulfil the demands. Wang and Smith (1994) developed PA to optimise RCN and developed Limiting Composite Curve (LCC). This technique was developed for targeting in RCNs with a single resource. Bandyopadhyay (2006) developed Source Composite Curve (SCC), another PA technique. SCC was applied to RCNs with multiple resources using Prioritized Cost, devised by Shenoy and Bandyopadhyay (2007). Prioritised Cost signifies an optimality criterion that decides whether another resource should be used. Prioritised Cost of a resource depends on its cost, quality, and Pinch Quality. Deng and Feng (2011) extended the application of LCC to RCNs with multiple resources using Prioritized Cost. PA techniques were applied to domains like carbon-constrained energy planning (Chandrayan and Bandyopadhyay, 2014) and water recycling networks (Chin et al., 2022). PA applications and techniques have considered deterministic parameters, but RCNs should also be designed to incorporate uncertainty. Flow and quality parameters in RCNs can be uncertain due to process variability, environmental factors, and supply chain disruptions. Uncertainty in parameters can be of two natures: Stochastic and Epistemic. Arya and Bandyopadhyay (2018) incorporated stochastic uncertainty in RCNs with multiple resources. Parameters are said to be epistemic uncertain if there is a lack of information about the system. Tan (2011) modelled epistemic uncertainty in RCNs using fuzzy programming. Further, Bandyopadhyay (2020) developed a model based on interval linear programming and PA using the Best-Worst approach. Application of this model was limited to single-resource RCNs. Existing literature has not yet discussed the targeting in RCNs with multiple resources and epistemic parameters.

This paper proposes a methodology based on the Best-Worst approach, PA and Prioritized Cost, to approach this problem. Several critical aspects of this work are identified while developing the methodology. The best and worst cases are identified for cost minimisation in the presence of multiple resources. Prioritized Cost formulation is modified to incorporate epistemic parameters. Limiting values for the quality interval of the second resource are determined using Prioritized Cost. Scenarios are generated by varying quality intervals of the second resource around these limiting values. Parameter values are identified at which the Prioritized Cost of the first and second resources is equal. The identified research gap is mathematically formulated in Section 2. The proposed methodology and the tools to apply it are explained in Section 3. A case study is presented in Section 4, which illustrates the methodology through the application.

2. Problem statement and mathematical formulation

The general structure of RCNs has four entities: sources, demands, resources, and waste. Figure 1 presents this system. Two parameters are associated with every entity: quality and flow. For RCNs, the quality is typically represented using an inverse numerical scale. It means that a higher numerical value indicates inferior quality. Under epistemic conditions, flow and quality are represented by interval numbers. Targeting of multiple resources in such problems can be mathematically defined as follows:

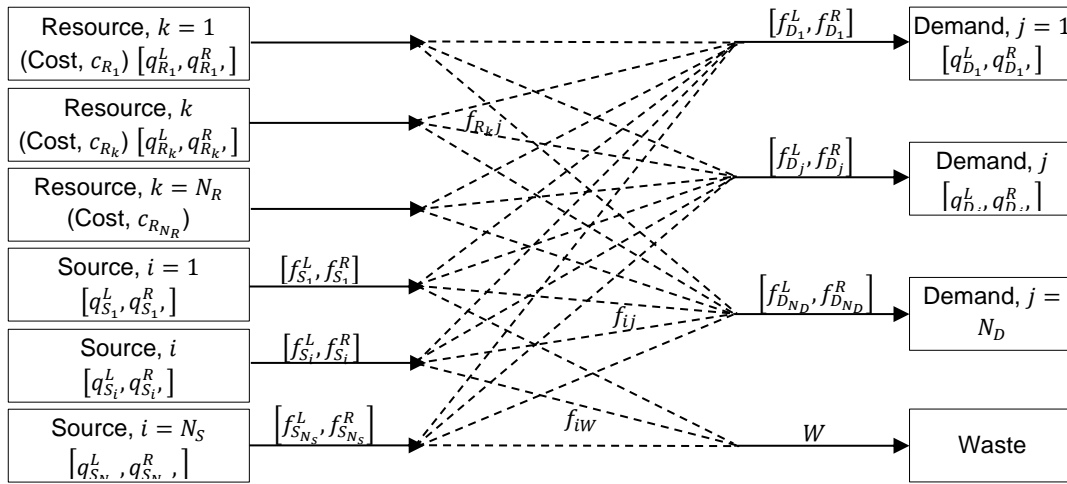


Figure 1: Graphical representation of resource conservation networks with epistemic parameters

- A set of N_S internal sources exist to provide streams with associated quality and flow. Quality of i_{th} source is defined by an interval number $[q_{S_i}^L, q_{S_i}^R]$. Similarly, the flow provided by i_{th} source is defined as $[f_{S_i}^L, f_{S_i}^R]$.
- A set of N_D internal demands need to be fulfilled with quality and flow. The maximum quality interval that j_{th} demand can accept is $[q_{D_j}^L, q_{D_j}^R]$. The flow provided to j_{th} demand must fall within given limits, i.e. $[f_{D_j}^L, f_{D_j}^R]$.
- A set of N_R resources exist, and each of them acts as an external source. These resources are also required to meet demands' quality and flow requirements. The quality interval of k_{th} resource is $[q_{R_k}^L, q_{R_k}^R]$. The cost of resources is considered deterministic in this problem. The cost of k_{th} resource denoted by is c_{R_k} .
- Waste is the last structural element of RCNs. All the unutilised flow is directed toward waste. It does not have any restriction on the flow and quality of the stream it can take and act as external demand.
- The objective is to optimise this structure for all the possible combinations of parameter values so that the total cost occurring due to resource utilisation can be minimised.

The mathematical formulation of the optimisation problem for RCN structure, presented in Figure 1, is as follows:

$$\text{Minimise, } R = \sum_{k=1}^{N_R} \left(c_{R_k} \times \sum_{j=1}^{N_D} f_{Rkj} \right) \quad (1)$$

Subjected to the following constraints:

$$\sum_{j=1}^{N_D} f_{ij} + f_{iw} = [f_{S_i}^L, f_{S_i}^R] \quad \forall i \quad (2)$$

$$\sum_{i=1}^{N_S} f_{ij} + \sum_{k=1}^{N_R} f_{Rkj} = [f_{Dj}^L, f_{Dj}^R] \quad \forall j \quad (3)$$

$$\sum_{i=1}^{N_S} f_{ij} [q_{S_i}^L, q_{S_i}^R] + \sum_{k=1}^{N_R} \left([q_{R_k}^L, q_{R_k}^R] \times \sum_{j=1}^{N_D} f_{Rkj} \right) \leq [f_{Dj}^L, f_{Dj}^R] \times [q_{D_j}^L, q_{D_j}^R] \quad \forall j \quad (4)$$

The objective function of this formulation is to minimise the total cost due to resource utilisation Eq.(1). This objective function is subjected to flow-balance constraints, that are, Eq.(2) and Eq.(3), and a quality-load balance constraint, i.e., Eq.(4). f_{ij} is the flow from source i to demand j , f_{iw} is defined as flow from source i to waste and f_{Rkj} is flow from resource k to demand j . $f_{S_i}^L$ and $f_{S_i}^R$ represent the lower and upper flow limit from the source i . The summation of all the flow from source i has to be within $f_{S_i}^L$ and $f_{S_i}^R$. Similarly, the total flow demand j can accept has to fall within $[f_{D_j}^L, f_{D_j}^R]$, where $f_{D_j}^L$ is the lower limit and $f_{D_j}^R$ is the upper limit of the flow for demand j . $q_{S_i}^L$ represents the lower limit of quality provided by source i , and $q_{S_i}^R$ is the upper limit of that quality. All the streams, from source i , must have quality value between $q_{S_i}^L$ and $q_{S_i}^R$. Similarly, all the flow streams from any resource k have quality in the range of $[q_{R_k}^L, q_{R_k}^R]$. Likewise, for any demand j , the maximum quality it can accept must be in the range $[q_{D_j}^L, q_{D_j}^R]$. Where $q_{D_j}^L$ is the lower limit and, $q_{D_j}^R$ is the upper limit of maximum quality any demand j can accept. This problem is solved using the Best-Worst approach and Prioritized Cost.

3. Solution methodology

This methodology is developed using the following tools: Best-Worst approach and Prioritized Cost. The discussion on these tools is as follows:

3.1 Best-Worst approach

This method identifies the best and worst conditions to optimise objective function by examining extreme parameter values (Shaocheng, 1994). The best conditions to minimise resource utilisation are those in which sources provide their purest quality and maximum flow while demands simultaneously require the minimum possible flow at the worst quality. The best conditions are the most favourable to minimise resource requirements. Conversely, the worst conditions are the most unfavourable for achieving minimum resource requirements. These occur when sources provide their worst quality and minimum flow, while demands require the purest quality and maximum flow; such conditions are called the worst conditions. The best-case solution can be defined as the minimum value attained through various parameter combinations. This implies that any value that falls below the best-case solution would be infeasible for any combination of source-demand parameters. Likewise, the worst-case solution is a value above which any solution is conservative but remains feasible in satisfying the demand. The worst-case solution represents the maximum of the minimum solutions across all possible parameter combinations.

Although the best and worst-case scenarios are often used to represent the problem, it is essential to address intermediate cases to attain a complete understanding. A predetermined variable λ is employed to address all cases through linear parametrisation. Tan (2011) proposed λ as a degree of fuzzy constraint satisfaction and optimized it using fuzzy linear programming. Eq.(5) shows the parametrization of flow from source i , as per the definition of the best-case and worst-case conditions.

$$\lambda = \frac{f_{S_i}^R - f_{S_i}}{f_{S_i}^R - f_{S_i}^L} \quad (5)$$

Similarly, intermediate values for other flow and quality parameters are represented through parametrisation. The best-case is represented by $\lambda = 0$, and the worst-case is represented by $\lambda = 1$. Pinch Analysis is used to optimise the problem under these defined conditions.

3.2 Prioritized cost

The Prioritized Cost is the optimization criterion for the utilisation of multiple resources, and it is calculated as the ratio of the resource cost to the difference between the Pinch Quality and resource quality. This notion is further extended to epistemic parameters by converting them into deterministic values via parametrization. For deterministic problems, the optimality criterion proposed by the Prioritized Cost concept is as follows:

$$\frac{c_{R_1}}{(q_p - q_{R_1})} > \frac{c_{R_2}}{(q_p - q_{R_2})} \quad (6)$$

Eq(6) implies that Prioritized Cost depends on pinch and resource quality. This concept dictates that the second resource would be introduced if its Prioritized Cost is lower than the first one. This optimality criterion is modified for epistemic parameters and mathematically formulated as follows:

$$\frac{c_{R_1}}{([q_{p,\lambda=0}, q_{p,\lambda=1}] - [q_{R_1}^L, q_{R_1}^R])} > \frac{c_{R_2}}{([q_{p,\lambda=0}, q_{p,\lambda=1}] - [q_{R_2}^L, q_{R_2}^R])} \quad (7)$$

Eq(7) can be developed into two extreme cases using the Best-Worst approach:

For Best-Case:

$$\frac{c_{R_2}}{(q_{p,\lambda=0} - q_{R_2}^L)} < \frac{c_{R_1}}{(q_{p,\lambda=0} - q_{R_1}^L)} \quad (8)$$

For Worst-Case:

$$\frac{c_{R_2}}{(q_{p,\lambda=1} - q_{R_2}^R)} < \frac{c_{R_1}}{(q_{p,\lambda=1} - q_{R_1}^R)} \quad (9)$$

Eq(8) and Eq(9) can be developed to provide limiting values for $q_{R_2}^L$ and $q_{R_2}^R$. These limiting values can be determined by using the following equations.

$$q_{R_2,limiting\ value}^L = q_{p,\lambda=0} - \frac{(q_{p,\lambda=0} - q_{R_1}^L)}{c_{R_1}/c_{R_2}} \quad (10)$$

$$q_{R_2,limiting\ value}^R = q_{p,\lambda=1} - \frac{(q_{p,\lambda=1} - q_{R_1}^R)}{c_{R_1}/c_{R_2}} \quad (11)$$

Different scenarios can be developed by varying the quality interval of the second resource around limiting values, determined through Eq(10) and Eq(11). The comparison of Prioritized Cost between single and multiple resources depends upon λ due to linear parametrisation of all cases. It is possible that at a certain λ the Prioritized Cost of both resources is equal. This λ can be determined by using the following equation:

$$\lambda = \frac{q_p^L(1 - CR) - q_{R_1}^L + q_{R_2}^L CR}{l_{q_p}(CR - 1) + l_{q_{R_1}} - l_{q_{R_2}} CR} \quad (12)$$

In Eq(12), CR is the cost ratio, defined as the ratio of the first resource cost to the second resource cost. l_{q_p} , $l_{q_{R_1}}$, $l_{q_{R_2}}$ represents the length of quality interval for pinch controlling source, first resource, and second resource. The necessary tools for resolving the problem have been discussed, and the proposed methodology involves several steps. First, the best and the worst conditions should be identified to minimize the total cost then extreme cases and intermediate cases are parametrised using λ . The second step is to apply the Pinch Analysis to solve the identified cases for the first resource and determine Pinch Quality. The third step is to assume the quality range of the second resource based on the limiting values of $q_{R_2}^L$ and $q_{R_2}^R$ and then the Prioritized Cost of the first and the second resources is compared for λ values. In the fourth step, the quality range of the second resource is varied around the limiting values to generate all possible scenarios. Finally, λ is determined for scenarios where the Prioritized Cost is equal for both resources. An example in the next section illustrates this methodology.

4. Illustrative example

Pharmaceutical industries use various solvents for different processes like absorption and degreasing. These solvents are hazardous upon exposure to the environment. Kazantzi et al. (2019) considered the hazardous nature of solvents as quality and minimised solvent requirements using Pinch Analysis. The data for this case study is adapted from Kazantzi et al. (2019) and comprises two parameters: Permissible Exposure Limit (PEL) and flow rate (kg/h). PEL is the minimum amount of chemical substance that poses a potential hazard upon exposure to surrounding. The lower PEL value indicates a higher risk due to exposure to chemical substance exposure. PEL value is reciprocated to define quality and make it suitable for the structure of the problem. Risk-Index (RI), also mentioned in the original data, is defined as the ratio of flow to the PEL and is equivalent to

quality load. Original data have precise parameters, but epistemic uncertainty is introduced by extending a limit to each parameter.

This case study involves two demands, an absorber and a degreaser, and four sources (heptane, hexane, isopropyl alcohol, methyl ethyl ketone) and a resource (acetone). The original case study has been extended to multiple resources by introducing another acetone resource whose quality range is variable. The quality parameters are made epistemic by providing a 5 % margin to the original values. Similarly, the margin assumed in flow parameters is 10 % of the original flow values. This assumption made the parameters suitable for the problem described in Section 2. The cost of resources is deterministic. For existing acetone, i.e., Resource 1, the cost is assumed to be 50 \$/kg and 16.67 \$/kg for Resource 2. All the values for the parameters are shown in Table 2.

Table 1: Flow and quality data for the example

Sources	Quality (1000/ppm)	Flow (kg/h)	Demands	Quality (1000/ppm)	Flow (kg/h)
Heptane	[1.9050 2.0000]	[1,000 1,100]	Absorber	[1.4286 1.5000]	[1,909 2,100]
Hexane	[2.1160 2.2220]	[920 1,012]	Degreaser	[2.5000 2.6250]	[1,545 1,700]
Isopropyl Alcohol	[2.3810 2.5000]	[760 836]			
Methyl Ethyl Alcohol	[4.7620 5.0000]	[210 231]			
Acetone1 (Resource 1)	[0.9524 1.0000]	-			
Acetone2 (Resource 2)	-	-			

The proposed methodology is applied to the presented data. Heptane is identified as pinch controlling source, and its quality range represents the pinch quality interval ([1.9050, 2.0000]). The limiting values determined for the lower extremity and upper extremity of Acetone2 quality are 1.5875 and 1.6667. Now assuming q_{R2}^L and q_{R2}^R to be 1.4 and 1.8. Figure 2a illustrates the variation of Prioritized Cost for the assumed quality interval of Acetone2, and Figure 2b shows total cost is plotted against λ .

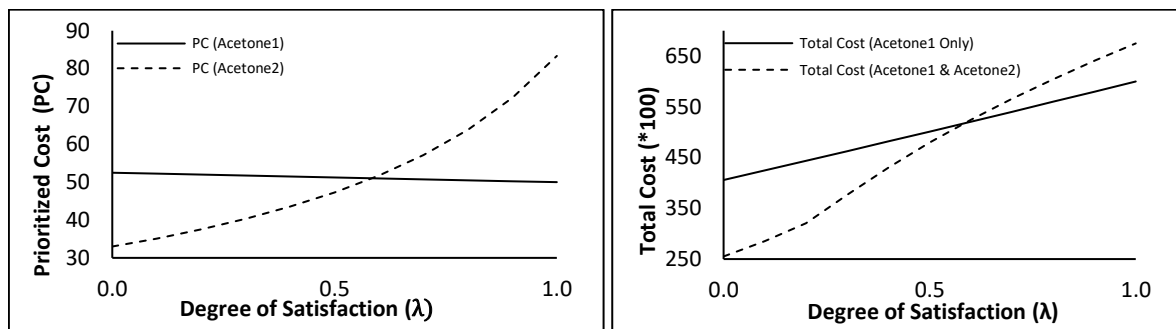


Figure 2: Variation in (a) Prioritized Cost (b) Total cost with respect to λ

Figure 2a displays the intersection of the Prioritized Cost curve for given resources. Using Eq(12), the intersection point is determined at $\lambda = 0.584$. For a value of λ less than 0.584, utilisation of Acetone2 is recommended, and vice-versa. Figure 2b suggests that under the best case, Acetone2 introduction reduces the total cost by 37.1 % from \$ 40,581 to \$ 25,516. Conversely, in the worst case, the total cost increases by 12.5 % from \$ 59,998 to \$ 67,498 if Acetone2 is introduced. This observation is developed as a scenario based on the quality interval assumption of Acetone2. Four such scenarios are possible if the quality interval of Acetone2 is varied around the limiting value for the Acetone2 quality interval. Scenario 1 is possible when Acetone1 is affordable for all values of λ . Scenario 2 is created when Acetone1 is affordable for the Best-Case, and Acetone2 is affordable for the Worst-Case. When Acetone2 is affordable for the Best-Case, and Acetone1 is affordable for the Worst-Case, it creates Scenario 3. If Acetone2 is affordable for all values of λ then Scenario 4 is possible. All these scenarios, presented in Figure 3, are developed by making various assumptions about the quality interval of Acetone2. Scenario 1 is possible if q_{R2}^L is more than 1.5875 and q_{R2}^R is more than 1.6667. If the quality interval of Acetone2 is between 1.5875 and 1.6667, then Scenario 2 is possible. The case presented in Figure 2 is shown as Scenario 3 in Figure 3. Scenario 2 also has an intersection point at $\lambda = 0.630$ like Scenario 3. However, Scenario 1 and Scenario 4 will not have such intersections as one resource is affordable for all values of λ . Scenario 4 is possible if the Acetone2 quality interval is such that q_{R2}^L is less than 1.5875 and q_{R2}^R is also less than 1.6667. These scenarios provide complete information about the Acetone2 resource and assist the designer in making an informed decision about its introduction to the existing RCN.

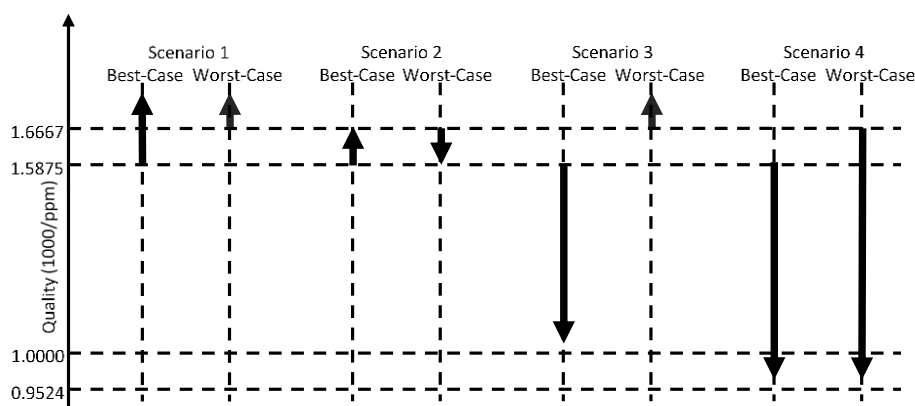


Figure 3: Pictorial representation of Acetone2 quality interval for all four possible scenarios

5. Conclusion

Resource conservation in process industries leads to overall sustainability through social, economic, and environmental benefits simultaneously. Pinch Analysis is a technique for resource conservation. In this paper, Pinch Analysis is applied to multiple resource RCN problems with epistemic parameters. A novel methodology is developed that compares the Prioritized Cost of resources under epistemic conditions. This methodology helps practitioners to minimize resource requirements based on risk taking ability to achieve overall sustainability without being conservative. The methodology was applied for solvent selection. Best-case conditions favoured introduction of Acetone2 and reduced the total cost by 37.1 % from \$ 40,580 to \$ 25,516. Acetone2 should not be selected under worst-case conditions. In this work, specific combination of parameter values are identified where the Prioritized Cost of both resources is equal and it is observed at $\lambda = 0.584$. Future work can include the incorporation of Pinch jump in the developed methodology, which can provide a more generalised approach for uncertainty assimilation in RCN designs. The presented work considered a deterministic cost of resources. In the future, the epistemic nature of cost will be considered.

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