

# Pinch Analysis-Based Targeting Multiple Resources in Water Conservation Networks

Mukkath Udayan Ashna\*, Santanu Bandyopadhyay

Department of Energy Science and Engineering, Indian Institute of Technology Bombay, Powai, Mumbai 400076, India  
19i170006@iitb.ac.in

Freshwater is a limited and irreversible resource that is essential for human existence. Freshwater conservation is one of the critical components for sustainable development, socioeconomic development, a healthy ecosystem and social survival. Pinch Analysis, a tool in process integration, can be used to conserve freshwater resources in water conservation networks (WCNs). In many WCNs, multiple freshwater resources exist; the primary objective is cost-effectively conserving freshwater. Typically, WCN problems with up to two freshwater resources are efficiently solved using the existing methodologies in Pinch Analysis. In WCNs with three or more freshwater resources, replacing some freshwater resources with a combination of other freshwater is possible and may lead to a cost-optimal solution. This replacement strategy will be demonstrated for the first time in this paper. This paper aims to develop an algorithm based on Pinch Analysis to identify the replacement criteria for freshwater resources. The proposed algorithm is graphically represented with the help of a cost vs. concentration sensitivity diagram. The proposed algorithm is explained through a numerical example.

## 1. Introduction

The increasing global population and economic shift amplified the demand for freshwater in various sectors, including industry, agriculture, etc., stressing the global freshwater resources. Such a rise in global freshwater intake has depleted over half of the world's largest aquifers. According to global water statistics, water covers 71 % of the earth's surface, and oceans constitute 96.5 %, implying that a significant part of these water resources is unfit for human intake (Domingo, 2012). The rising population and global average temperature change the dynamics between the demand and supply of freshwater. Global freshwater demand is expected to increase by 55 % by the end of 2050 (Colella et al., 2021). Among the total demand, 70 % accounts for agriculture, whereas the freshwater requirement for energy generation is projected to increase by 20 % (Dikshit and Choukiker, 2005). Most freshwater resources are limited in the current environment and are expected to be at a scarcity level immediately. The tension created because of freshwater scarcity affects the effective use of water bodies worldwide, creating other issues on social, cultural, economic, and political aspects of society. To solve water scarcity and achieve sustainability, United Nations Sustainable Development Goals (SDGs) target 6.4 aims that sustainability will boost water usage effectiveness across all sectors by 2030 and ensure sustainable withdrawals and supply of freshwater (UNSDG, 2018). So the optimised use of the existing freshwater resources can serve to some extent as an immediate solution to the problems. Pinch analysis, a widespread technique in process integration, is considered an efficient approach to optimising resource conservation networks (RCNs) problems.

In the 1970s, the concept of Pinch analysis was developed by Linnhoff as a thermodynamically based tool for targeting and designing heat recovery systems (Linnhoff and Hindmarsh, 1983). In 1994 the mass and heat integration framework was extended to study wastewater minimisation in water networks by Wang and Smith (1994). The objective of the RCNs problems belonging to the water domain is to minimise the freshwater requirements accordingly (Agrawal and Shenoy, 2006). Two major graphical methods, Source Composite Curve (SCC) and Limiting Composite Curves (LCC) (Bandyopadhyay, 2015), are used to address RCN problems. The methodology for targeting multiple freshwater resources through source composite curves has

already been established by Shenoy and Bandyopadhyay (2007) using the concept of prioritised cost. Foo (2007) has addressed problems with single and multiple freshwater utilities using the numerical targeting tool of water cascade analysis. Taking inspiration from targeting multiple freshwater resources, Alwi and Manan (2007) have established a new methodology based on source and sink composite curves to find the minimum requirement of various freshwater resources. The total cost minimisation problem in a batch process using multiple freshwater resources was mathematically solved by Chaturvedi and Bandyopadhyay (2014).

Through a motivational problem from the water management domain, it is found that Pinch Analysis purely based on prioritised cost is insufficient to produce the optimal solution for problems with multiple freshwater resources. It is identified that some resources can be replaced with a combination of other available resources to have a cost-effective solution. This paper aims to determine the missing criterion in finding the optimal solution for targeting multiple freshwater resources using Pinch analysis. By identifying the criterion, the following novelties can be introduced.

- The mathematical formulation for the replacement criterion for targeting multiple freshwater resources is established.
- A schematic representation of sensitivity analysis between cost and resource quality is studied thoroughly to understand the mixing rule of freshwater resources.

## 2. Problem statement

The RCNs problems from the water management domain with multiple freshwater resources in Pinch Analysis can be generally explained as follows.

There are total  $N_s$  existing water sources, and each of these sources produce a constant water flow of  $F_s$  with contaminant concentration  $q_s$ . Total  $N_d$  demands are present, which need to be satisfied with the sources and freshwater. Each demand is characterised in such a way that every demand can accept a flow  $F_d$  with a maximum acceptable contaminant concentration limit of  $q_d$ . Along with the sources total  $N_r$  freshwater resources are available to supply the  $N_d$  demands. The freshwater resources do not have any flow restrictions. Each of the resources is portrayed with an impurity concentration  $q_{rs}$  and cost  $C_{rs}$ . The unused flow from sources is considered as wastewater. The requirement for freshwater can be identified by analysing the associated process data. Multiple freshwater resources may be used to produce a cost-effective solution for the entire process. The pictorial representation of the whole problem is shown in Figure 1.

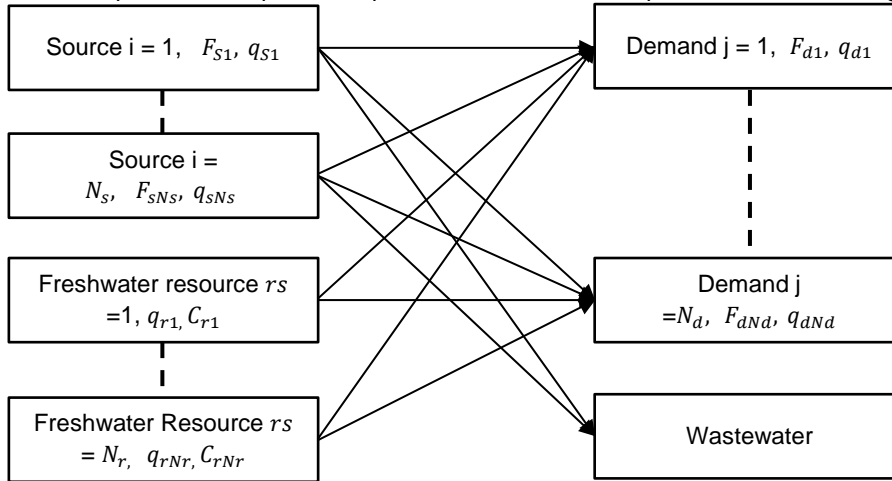


Figure 1: Schematic representation of resource allocation problem

The water flow balance equations for sources, demand, and contaminant load balance equations for demand are explained below. Let  $f_{ij}$  be the flow transferred from source  $i$  to demand  $j$ . The flow from source  $i$  to waste is symbolised as  $f_{iw}$ . The flow produced from resource  $rs$  to demand  $j$  is represented as  $f_{rsj}$ .

$$\sum_{j=1}^{N_d} f_{ij} + f_{iw} = F_{si} \quad \forall i \in N_s \quad (1)$$

$$\sum_{i=1}^{N_s} f_{ij} + \sum_{rs=1}^{N_r} f_{rsj} = F_{dj} \quad \forall j \in N_d \quad (2)$$

$$\sum_{i=1}^{N_s} f_{ij} q_{si} + \sum_{r=1}^{N_r} f_{rsj} \leq F_{dj} q_{dj} \quad \forall j \in N_d \quad (3)$$

The total operating cost of the water network may be represented as

$$\phi = \sum_{i=1}^{N_r} F_i C_i \quad (4)$$

The objective is to minimise Eq.(4) and obtain a cost-effective solution subject to the constraints from Eq.(1) to Eq.(3).

## 2.1 Prioritised cost and prioritising sequence

Prioritised cost (PC) is a significant component of Pinch analysis which helps to identify the cost-efficient solution for problems with multiple freshwater resources. Using prioritised cost, decision-makers can concentrate on optimising some specific freshwater bodies through which an optimal solution can be produced. The prioritised cost is a primary notion in targeting multiple freshwater resources and improving efficiencies. Overall prioritised cost helps the decision maker prioritise the work and increase efficiency to produce the optimal solution. The prioritised cost of each freshwater resource can be calculated using Eq(5).

$$PC_i = \frac{C_i}{q_p - q_i} \quad \forall i \in N_r \quad (5)$$

The freshwater resource is represented as  $FR_i$  where  $i$  ranges from 1 to  $N_r$ . The index number of the resource and its quality follows an inverse trend, as the index of the resource increases, the purity of the resource decreases. Eq(5) represents the optimality criterion for freshwater stream selection in RCNs problems where  $C_i$  and  $q_i$  are the cost and impurity concentration associated with freshwater resource  $FR_i$  and  $q_p$  is the Pinch quality of the problem. A resource  $FR_i$  will be part of the optimal solution only if it has a lower prioritised cost than the resources  $FR_1, FR_2, \dots, FR_{i-1}$ . A prioritising sequence is generated by arranging the resources in decreasing order of PC and increasing order of quality in a square bracket (Priya and Bandyopadhyay, 2017).

## 2.2 Motivational problem

Here, the problem statement is explained through a motivational problem. The process data for the problem is given below in Table 1 from the water management domain (Foo, 2009). Water flow in t/h is the flow variable, contaminant concentration in parts per million is the quality, and wastewater is the waste of the problem. The data for freshwater resources is given in Table 2. The purest resource  $FR_1$  has a quality (contaminant concentration) of 0 ppm with a cost of 100 \$/t,  $FR_2$  with a quality of 40 ppm at the cost of 40 \$/t and  $FR_3$  with a quality of 55 ppm and cost 10 \$/t. In the absence of resource data, consider the first resource as purer and the cost of it to be greater than the second resource, and this trend is maintained with the following resources as well.

Table 1: Process data for motivational problem from the water domain (Foo, 2009)

<u>Sources</u>	Quality (ppm)	Flow(t/h)	<u>Demands</u>	Quality (ppm)	Flow (t/h)
S1	100	20	D1	0	20
S2	100	100	D2	50	100
S3	800	40	D3	50	40
S4	800	10	D4	400	10

The Pinch Point of the problem is found to be 100 ppm using the limiting composite curve in Figure 2.

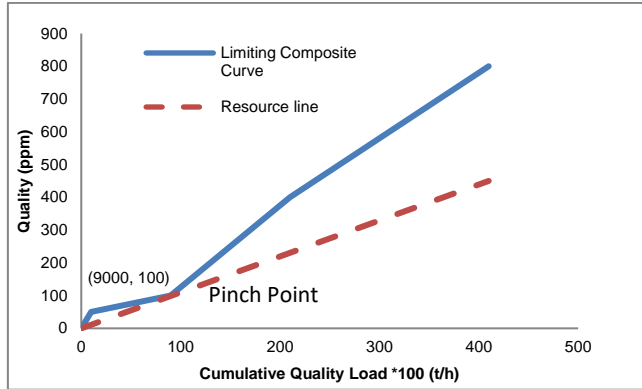


Figure 2: Limiting composite curve of the motivational problem from the water domain

The prioritised cost of three resources is calculated using Eq(5) and tabulated in Table 2.

Table 2: Data and Prioritised cost of resources

Resource	Quality (ppm)	Cost (\$/t)	Prioritised cost
FR1	0	100	1
FR2	40	40	0.67
FR3	55	10	0.22

It is observed that the prioritised costs of three resources follow the optimality criteria given in Eq(5), that is  $PC_1 > PC_2 > PC_3$  and conferring to this, it is anticipated that the prioritising sequence to be  $[FR_1 - FR_2 - FR_3]$ . According to the prioritised cost trend, the optimal solution is 4,800 \$/t with a breakdown of 20 t/h from  $FR_1$ , 46.67 t/h from  $FR_2$  and 93.33 t/h from  $FR_3$ . But verification shows that the solution obtained purely based on prioritised cost is not optimal. The optimal solution for the given problem is 4,545.55 t/\$ with a breakdown of 32.72 t/h from  $FR_1$  and 127.27 from  $FR_3$ , which is 5.3 % less than the solution only based on prioritised cost. Analysis of both solutions shows that Pinch analysis centred only on prioritised cost fails to produce the optimal solution. In this problem, water flow produced by resource  $FR_2$  is replaced by the water flow produced by both  $FR_1$  and  $FR_3$  to produce a cost-effective solution. The reason for the replacement of  $FR_2$  is unknown, even if it follows the optimality criteria. This paper targets to find a generalised mathematical formulation for the replacement criterion of resources to produce cost-effective optimal solutions in RCNs problems. A cost Vs concentration diagram is introduced to graphically establish the optimality and replacement criteria for resource selection.

### 3. Mathematical formulation

Consider the situation where only the purest resource  $FR_1$  is accessible to supply for the demand. It is possible that a cost-effective solution can be attained by providing a portion of demand with another accessible resource  $FR_2$  this resource which in turn can be replaced by a third resource  $FR_3$ . This replacement can occur in two different scenarios according to the optimality condition. Two resources  $FR_1$  and  $FR_2$  is having quality  $q_1$  and  $q_2$  and cost  $C_1$  and  $C_2$ . Since  $FR_1$  is purer than  $FR_2$ ,  $q_1 < q_2$  and  $C_1 > C_2$ . Adding a second resource  $R_2$  is cost-optimal only if

$$\frac{C_1}{q_p - q_{r1}} > \frac{C_2}{q_p - q_{r2}} \quad (6)$$

The third resource  $FR_3$  is considered with quality  $q_3$  and cost  $C_3$ . The trend of the qualities and costs are as  $q_1 < q_2 < q_3$  and  $C_1 > C_2 > C_3$ .

#### 3.1 Case 1 - $PC_1 < PC_2$ and $PC_1 > PC_3$

Eq.(5) suggests that lower prioritised cost leads to optimality. Case 1 shows that the resource  $FR_2$  is not following the optimality criterion but both  $FR_1$  and  $FR_3$  follow it. The conditions essentially indicate that  $FR_2$  will not be part of the optimal solution where both  $FR_1$  and  $FR_3$  only constitute the optimal solution creating a prioritising sequence  $[FR_1 - FR_3]$ .

### 3.2 Case 2 - $PC_1 > PC_2 > PC_3$

The resource data provided falls under the optimality condition explained in Eq.(5). The optimality condition alone says that resources  $FR_1$ ,  $FR_2$  and  $FR_3$  will be part of the optimal solution creating a sequence  $[FR_1 - FR_2 - FR_3]$ . It is observed that the optimality condition alone is not sufficient to produce the optimal solution. Resource  $FR_2$  will only be part of the optimal solution if the cost of the flow produced by  $FR_2$  alone is less than the cost of the same amount of flow produced by both  $FR_1$  and  $FR_3$  together. If the cost of flow produced by  $FR_2$  is greater than that of the same amount of flow produced by both  $FR_1$  and  $FR_3$  together then  $FR_2$  will not be part of the optimal solution and, eventually  $FR_2$  will be replaced with  $R_3$  creating a prioritising sequence  $[FR_1 - FR_3]$ . This replacement condition can be mathematically formulated as

$$C_2 > \frac{C_1 (q_3 - q_2) + C_3 (q_2 - q_1)}{q_3 - q_1} \quad (7)$$

Eq.(7) establishes the condition for replacing  $FR_2$ . By considering the three preceding resources  $FR_{i-1}$ ,  $FR_i$  and  $FR_{i+1}$  with quality  $q_{i-1}$ ,  $q_i$  and  $q_{i+1}$  Eq.(7) can be generalised as

$$C_i > \frac{C_{i-1} (q_{i+1} - q_i) + C_{i+1} (q_i - q_{i-1})}{q_{i+1} - q_{i-1}} \quad (8)$$

The optimality and replacement criteria can be graphically drawn, and the prioritising sequence can be found efficiently. The applicability of the criterion is explained through an example from the water domain in the next section.

## 4. Solved problem

The explained methodology is applied to the problem given in Table 1. Through this solved example, the proposed criterion is established graphically, for that the third resource  $FR_3$  is considered with varying cost and quality. Using the resource data of  $FR_1$  and  $FR_2$  and Pinch quality, the sensitivity analysis diagram is drawn in Figure 3a.

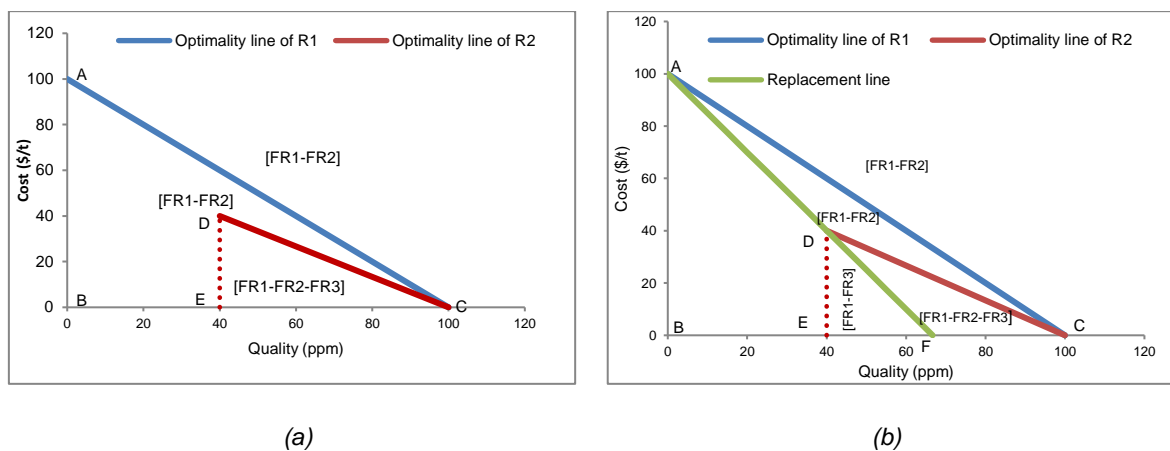


Figure 3: (a) Sensitivity analysis diagram with  $R_1$  and  $R_2$  and (b) Sensitivity analysis diagram with the replacement line

From Figure 3a, it can be analysed that if  $FR_2$  is taking any cost and quality inside the triangle ABC, the resulting solution will consist of  $FR_1$  and  $FR_2$ . Since the given resource data of  $FR_2$  (quality 10 ppm, cost 60 \$/t) is in the prescribed region, the optimal solution is  $[FR_1 - FR_2]$ . Figure 3a also has two optimality lines, producing two triangles, ABC and DEC. The Triangle DEC primarily gives optimality criteria for  $FR_3$  data selection. Figure 3b essentially says that if  $FR_3$  takes any cost and quality inside triangle DEC, then only  $FR_3$  will be part of the optimal solution. Any combination of cost and quality for  $FR_3$  outside the triangle, DEC will give a prioritised cost of  $FR_3$  as greater than purer resources to it, not being a part of the optimal solution. According to the proposed criteria, the prioritised cost and replacement condition must be simultaneously considered to get the optimal solution. Figure 3b represents the sensitivity analysis diagram with optimality lines of  $FR_1$  and  $FR_2$  along with the replacement line that passes through  $FR_1$  and  $FR_2$ . The replacement line divides the triangle DEC into two, triangles DEF and DFC. If  $FR_3$  takes a data point inside the triangle DEC,

then  $FR_2$  will be replaced with the combination of  $FR_1$  and  $FR_3$ , resulting solution in this region as  $[FR_1 - FR_3]$ . Similarly, if the credentials of  $FR_3$  is inside the triangle DFC then the solution will be  $[FR_1 - FR_2 - FR_3]$ . The results in this example are verified using solver in Excel.

## 5. Conclusions

This paper proposes a replacement criterion for targeting multiple freshwater resources using Pinch analysis. This paper establishes a new strategy for replacing a particular freshwater resource with a combination of other resources to obtain a cost-optimal solution mathematically and graphically. The concept is built upon the optimality criteria prioritised cost in Pinch analysis. The existing methodologies were unaware of this replacement strategy which gives complete information about the optimal solution. Neglecting the replacement criteria might lead the decision maker to a wrong optimal solution. The cost-optimal solution changes with the cost and quality of the third resource. For example, keeping the data for  $R_1$  and  $R_2$  as given and 54 ppm as the quality and 5 \$/t as the cost for  $R_3$ , the optimal solution is 4,100 \$/t according to the existing method. Still, by utilising the developed replacement criterion, the optimal solution obtained is 3,685.19 \$/t which is 10.11 % lesser than the solution attained without considering the replacement criterion. Using the design method proposed in this paper, an optimal solution can be achieved accurately without being misled. Also, the established criterion is efficient in assisting the decision-maker in selecting resource parameters according to the prioritising sequence. The recommended methodology is sufficient to solve the RCN problems from various domains like water, hydrogen, etc. Future work can be considered with RCN problems with more than four resources, creating a complicated sensitivity analysis diagram.

## References

- Agrawal V., Shenoy U.V., 2006, Unified Conceptual Approach to Targeting and Design of Water and Hydrogen Networks. Willey Interscience, 52(3), 1071–1082.
- Alwi S.R.W., Manan Z.A., 2007, Targeting multiple water utilities using composite curves. Industrial and Engineering Chemistry Research, 46(18), 5968–5976.
- Bandyopadhyay S., 2015, Mathematical Foundation of Pinch Analysis. Chemical Engineering Transactions, 45, 1753–1758
- Chaturvedi N.D., Bandyopadhyay S., 2014, Optimisation of multiple freshwater resources in a flexible-schedule batch water network. Industrial and Engineering Chemistry Research, 53(14), 5996–6005.
- Colella M., Ripa M., Coccozza A., Panfilo C., Ulgiati S., 2021, Challenges and opportunities for more efficient water use and circular wastewater management. The case of Campania Region, Italy. Journal of Environmental Management, 297, 113117.
- Dikshit A.K., Choukiker S.K., 2005, Global Water Scenario: the Changing Statistics, Vision RI Research Links, 236, 540–549.
- Domingo M.C., 2012, An overview of the internet of underwater things. Journal of Network and Computer Applications, 35(6), 1879–1890.
- Foo D.C.Y., 2007, Water cascade analysis for single and multiple impure fresh water feed. Chemical Engineering Research and Design, 85(8 A), 1169–1177.
- Foo D.C.Y., 2009, State of the art review of pinch analysis techniques for water network synthesis. Industrial and Engineering Chemistry Research, 48(11), 5125–5159.
- Linnhoff B., Hindmarsh E., 1983, The pinch design method for heat exchanger networks. Chemical Engineering Science, 38, 745–763.
- Priya G.S.K., Bandyopadhyay S., 2017, Multiple objectives Pinch Analysis. Resources, Conservation and Recycling, 119, 128–141.
- Shenoy U.V., Bandyopadhyay S., 2007, Targeting for Multiple Resources. Industrial and Engineering Chemistry Research, 46, 3698–3708.
- UNSDG 2018. United Nations, Sustainable Development Goal 6 Synthesis Report 2018 on Water and Sanitation, United Nations, New York.
- Wang Y.P., Smith R., 1994, Wastewater minimisation. Chemical Engineering Science, 49(7), 981–1006.