

A Disjunctive Programming Approach for Sustainable Design of Municipal Solid Waste Management

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The increasing Municipal Solid Waste (MSW) poses great pressure on cities, especially for emerging countries. This paper presents a mathematical programming model to optimise MSW management. A disjunctive fuzzy optimisation approach is used in this work for several MSW treatment technologies, resulting in multi-objective optimization. The proposed model is used to choose the suitable treatment technologies and optimal distribution of MSW between sources and sinks to minimise the total cost of MSW management and greenhouse gas emission. The optimisation problem is formulated as a mixed-integer linear programming model using generalised disjunctive programming to select suitable MSW treatment technologies and optimal distribution networks. A case study of Hefei, China is presented to illustrate the proposed approach. The multiple objectives of total cost and GHG emissions for MSW management are analysed based on fuzzy optimisation yielding a compromise solution of 192.99 M\$/y for total cost and 1,120.48 t/y for emissions. A compromised solution suggested in this study can effectively minimize both total cost and carbon emission and properly deal with the MSW management problem. This work provides a decision tool for the selection of MSW technologies during MSW management.

1. Introduction

The rapid urbanization and population growth have led to an increase in the generation of MSW. The increasing production of MSW is a serious challenge for local governments. In 2020, about 235 Mt of MSW were collected in China (Li et al., 2022). Among the MSW, about 21 % was used in sanitary landfills, 72.5 % through waste incineration and the remaining 6.5 % by biological processes. Improper disposal of MSW poses significant health hazards and environmental pollution risks, especially in densely populated areas. Sustainable MSW management strategies are needed to tradeoff the cost of treatment and the environmental impact.

MSW is composed of various materials, including organic waste, plastics, paper, glass, metals, and hazardous waste (Pattnaik and Reddy, 2010). These materials have different physical and chemical properties and are treated by different technologies. The composition of MSW differs for different cities and different countries. The basic steps in MSW management are: (i) generation of wastes at the source; (ii) collection and transfer of waste; and (iii) disposal, processing and treatment of waste (Nanda and Berruti, 2021). A systematic framework is required to guarantee the reliability and efficiency of MSW management systems by optimising the match between the waste sources and treatment technology. Incineration is a thermochemical technology that converts waste into energy (Abbasi et al., 2022). It is widely deployed all over the world because of its significant waste reduction and more energy production (Nanda and Berruti, 2020). However, there are also concerns about the air pollutants produced during the incineration process (Cudjoe and Acquah, 2021). Compared to incineration, landfilling is advantageous with low investments and less labor-intensive procedures (Sondh et al., 2022). Landfilling can generate harmful Greenhouse Gases (GHGs), commonly known as "Landfill Gases" (LFGs), such as methane which contribute to climate change (Siddiqua et al., 2022).

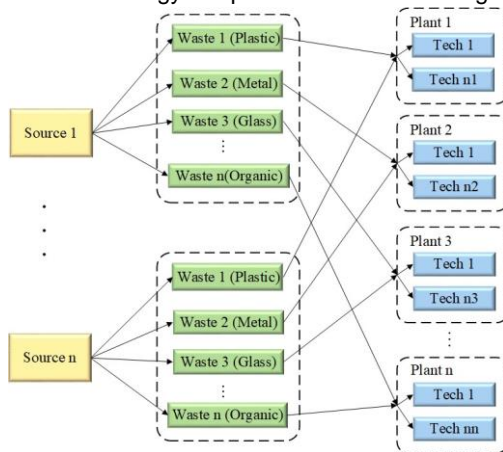
Mathematical programming methods are widely used to optimise MSW management and choose suitable technologies and treatment locations. For example, Ooi and Woon (2021) developed a multi-objective Mixed-

Integer Linear Programming (MILP) approach to determine the optimum allocation of MSW in different disposal and treatment facilities. The augmented ϵ -constraint method is used to solve the proposed model. Li et al. (2022) proposed crisp and fuzzy models for optimal waste management network synthesis and considered the uncertain parameters for MSW management. Zhao et al. (2023) developed MINLP and MILP models to design the logistics distribution strategy and facility expansion scheme for urban solid waste treatment in Qingdao. However, their study ignored the optimal match between wastes and MSW treatment technologies. Obuobi et al. (2022) analysed the electricity generation prospects of MSW with LFG to energy project and anaerobic digestion project. They also evaluated the economic viability of these projects. Saif et al. (2022) proposed a dynamic discrete optimisation model for MSW treatment with a sustainability objective. Their research considered the social targets and environmental impacts of technology and transportation. Deng et al. (2022) presented a multi-objective optimisation model to explore the optimal operational strategy for food waste using anaerobic co-digestion and transesterification. The economic, environmental, and social objectives are considered simultaneously. Adiansyah (2022) compared carbon emission reduction of three different waste management strategies based on life cycle assessment. Chee et al. (2022) proposed a non-linear programming model to allocate organic waste resources and satisfy the carbon-to-nitrogen ratio among the sources and sinks. Most of the aforementioned work focused on the optimisation of the MSW supply chain network and chose the treatment technologies by employing the binary integer in the model. It results in a large model and consumes computational resources when there are many MSW sources and sinks. In this work, a superstructure-based disjunctive programming model is proposed to design an optimal supply chain network and suitable treatment technologies from economic and environmental perspectives. The selection of the technologies is realised by the logic condition in the model. Less binary variables are involved in the model. The optimisation problem is finally formulated as a MINLP model using the big-M reformulation method. A case study of MSW management in Hefei, China, is used to demonstrate the application of the proposed models.

2. Problem statement

The problem addressed in this paper involves the selection of MSW treatment technologies and the allocation of MSW from sources to sinks. The objective is to choose the optimal MSW supply chain to minimise the total cost and environmental impact. The given information on the problem is shown below.

- A set of waste sources or transfer stations $i \in I$. Each source has different types of waste $w \in W$. The total amount of waste at each source and its composition is known.
- A set of waste sinks or disposal sites $j \in J$. Each sink has a maximum capacity for MSW disposal. The distances between waste sources and treatment sinks are known.
- A set of waste treatment technologies $k \in K$. The cost function and carbon emission accounting of each treatment technology are provided. The technologies in each sink are also known.



● *Figure 1: The superstructure of MSW management*

3. Model formulation

Figure 1 shows the superstructure for MSW management, which involves the allocation of waste from sources to sinks and the deployment of treatment technologies at sinks. The waste collected at source i is sorted and sent to the treatment plant j . In each treatment plant, a specific technology can be chosen to treat specific waste.

The selection of the treatment technology is represented by the disjunctive term. For a specific treatment plant, the number of technologies for waste disposal is constrained.

3.1 Optimization model

Every source of waste has its distinct composition. It is possible to segregate waste into various categories such as plastic, wood, textile, paper, and food. Eq(1) defines the maximum flowrate ($F_{i,w}^{Max}$) of waste w at source i , which is determined by the amount of waste collected (F_i) at source i and its composition factor ($x_{i,w}$). Eq(2) indicates the available flowrate of waste w at source i because of the MSW classification. The available flowrate of waste w at source i ($F_{i,w}^{Ava}$) is a portion of the total production of waste w at source i . The unseparated wastes are treated by incineration or landfilling. The flowrate of unseparated wastes (F_i^{Mix}) is determined by Eq(4).

$$F_i * x_{i,w} = F_{i,w}^{Max}, \forall i \in I, w \in W \quad (1)$$

$$F_{i,w}^{Max} * \alpha = F_{i,w}^{Ava}, \forall i \in I, w \in \{paper, food, plastic\} \quad (2)$$

$$F_{i,w}^{Max} + F_i^{Mix} = F_{i,w}^{Ava}, W = \{others\} \quad (3)$$

$$F_i^{Mix} = \sum_{w \in W} F_{i,w}^{Max} * (1 - \alpha), \forall i \in I, W = \{paper, food, plastic\} \quad (4)$$

The amount of separated waste w at source i is the sum of the amounts of the waste w transferred from the source i to all the sinks, as given by Eq(5). $F_{i,j,w}$ represents the amounts of the waste w transferred from the source i to the treatment sink j . It is assumed that wastes transferred to the sink are treated by facilities and are not stored. Eq(6) defines the amounts of waste w in the sink j is the sum of the waste treated by all the technologies. The amount of the wastes treated by the technology k at sink j is presented in Eq(7).

$$F_{i,w}^{Ava} = \sum_{j \in J} F_{i,j,w}, \forall i \in I, w \in W \quad (5)$$

$$F_{i,j,w} = \sum_{k \in K} F_{i,j,w,k}, \forall j \in J, w \in W \quad (6)$$

$$F_{j,k} = \sum_{i \in I} \sum_{w \in W} F_{j,w,k}, \forall j \in J, k \in K \quad (7)$$

Eq(8) represents a set of disjunctions for the selection of the technology at sinks, $j \in J$, each of which contains D_j terms. Each term of the disjunction has a Boolean variable $Y_{j,k}$ and an associated set of inequalities and equalities. Eq(9) indicates exactly one of the Boolean variables can be selected in each disjunction. For an active term ($Y_{j,k} = True$), the corresponding inequalities and equalities are enforced. Otherwise, the corresponding constraints are ignored. $ENI_{j,k}$ and $ECl_{j,k}$ represent environmental and economic indicator of each technology. Eq(9) is the Boolean variable.

$$\forall_{k \in D_j} \left[\begin{array}{l} Y_{j,k} \\ F_{j,k}^L \leq F_{j,k} \leq F_{j,k}^U \\ ENI_{j,k} = GHG_{j,k} \\ ECl_{j,k} = INV_{j,k} + OM_{j,k} \end{array} \right] \vee \left[\begin{array}{l} \neg Y_{j,k} \\ F_{j,k} = 0 \\ ENI_{j,k} = 0 \\ ECl_{j,k} = 0 \end{array} \right], j \in J \quad (8)$$

$$\forall_{k \in D_j} Y_{j,k}, Y_{j,k} = \{True, False\}, j \in J, k \in D_j \quad (9)$$

GHG emissions from transportation, landfill, incineration, anaerobic digestion and composting are all taken into consideration in this section. Eq(10) describes the GHG emission (GHG_{trans}) from MSW transportation.

$$GHG_{trans} = \sum_i \sum_j \sum_w \frac{F_{i,j,w} S_{i,j}}{n\eta} (E_{CO_2}^{trans} + 25E_{CH_4}^{trans} + 298E_{N_2O}^{trans}) + \sum_i \sum_w \beta F_{i,w}^{Ava} E_{press} \quad (10)$$

Where n is the load capacity of the vehicles. which is determined by the total amount of waste and the load capacity of the vehicle. $S_{i,j}$ is the distance between source i and sink j . η is the fuel efficiency. $E_{CO_2}^{trans}$, $E_{CH_4}^{trans}$ and $E_{N_2O}^{trans}$ represent the carbon dioxide, methane and nitrous oxide emission from diesel consumption per liter. β is the CO_2 emission coefficient of the electricity. E_{press} is the energy consumption for garbage compressed.

For the detailed models for GHG emission accounting of MSW treatment, please refer to Du et al. (2022). The detailed models for the investment costs and corresponding operating cost of each technology refer to Tsilemou and Panagiotakopoulos (2006). The capacity ranges from 20,000 to 200,000. Eq(11) and Eq(12) represents the cost during MSW transportation and sorting process. $Cost_{diesel}$ represent the cost of diesel per liter. $Sort_{cost}$ is the unit cost of waste during sorting process.

$$COST_{trans} = 300 \times \sum_i \sum_j \sum_w \frac{F_{i,j,w} S_{i,j}}{n\eta} \times Cost_{diesel} \quad (11)$$

$$COST_{sorting} = Sort_{cost} \times \sum_i \sum_w F_{i,w}^{Ava} \times 300, \forall i \in I, w \in \{paper, food, plastic\} \quad (12)$$

The GHG emissions of MSW management system is presented in Eq(13) and the total cost of the system is shown in Eq(14). The fuzzy optimization is used to find the compromise solution between GHG emissions and

the cost (Li et al., 2022). The fuzzy constraints are shown in Eqs(15) and (16). The objective function is presented in Eq(17). It is to maximize the degree of satisfaction (γ), ranging from 0 (unsatisfactory) to 1 (entirely satisfactory). A higher value is desired for the degree of satisfaction if the problem is to simultaneously satisfy GHG emissions and total cost. The problem is reformulated as MINLP model based on Big M reformulation.

$$ENI = GHG_{trans} + \sum_j \sum_k ENI_{j,k} \quad (13)$$

$$ECI = \sum_j \sum_k CCOST_{j,k} + \sum_j \sum_k OCOST_{j,k} + COST_{trans} + COST_{sorting} \quad (14)$$

$$ENI \leq ENI^U - \gamma(ENI^U - ENI^L) \quad (15)$$

$$ECI \leq ECI^U - \gamma(ECI^U - ECI^L) \quad (16)$$

$$Max \gamma \quad (17)$$

4. Case study

A case study of MSW management in Hefei, China is used to illustrate the proposed approach. In 2020, Hefei had a population of 9.37 million, with an urban population of 7.75 million. The amount of MSW collected in Hefei was 2.71 Mt (Hefei Municipal Statistics Bureau, 2021). The data for the case study are shown in Table 1. It includes the waste generation and the distance for transportation. The composition for food, paper, plastic, and other on average is 49.64 %, 12.80 %, 27.45 %, and 10.11 % (Du et al., 2022).

Table 1: Waste source data for the case study

Source	Waste generation t/d	Distance to sinks (km)						
		SK1	SK2	SK3	SK4	SK5	SK6	SK7
SR1: Shushan	1,030	34	28	26	26	38	24	21
SR2: Jingkai	530	38	32	23	23	57	33	13
SR3: Gaoxin	270	51	45	24	24	42	14	30
SR4: Baohe	1,360	24	18	37	37	50	42	10
SR5: Yaohai	890	25	19	38	38	43	40	12
SR6: Xinzhan	380	32	26	37	37	34	36	17
SR7: Luyang	1,100	41	35	37	37	31	35	26
SR8: Feixi	930	50	43	10	10	60	26	30
SR9: Feidong	850	20	14	54	54	50	54	25
SR10: Chaofeng	900	98	100	106	106	43	100	90
SR11: Chaohu	700	45	42	88	88	103	110	56
SR12: Lujiang	720	91	86	60	60	113	85	82

4.1 Scenario 1: emissions minimization

The objective is to minimize the GHG emissions of MSW transport and treatment process. The minimum GHG emissions are determined to be 1,119.66 t/y, of which the transport emissions account for 6 %, and the treatment emissions 94 %. The corresponding total cost is 193.53 M\$/y. The transport and sorting costs are 6.7 and 39.08 M\$/y. A sorting coefficient of 0 – 1 is examined to explore the effect of waste sorting coefficient on the total cost and GHG emissions, as shown in Figure 2a.

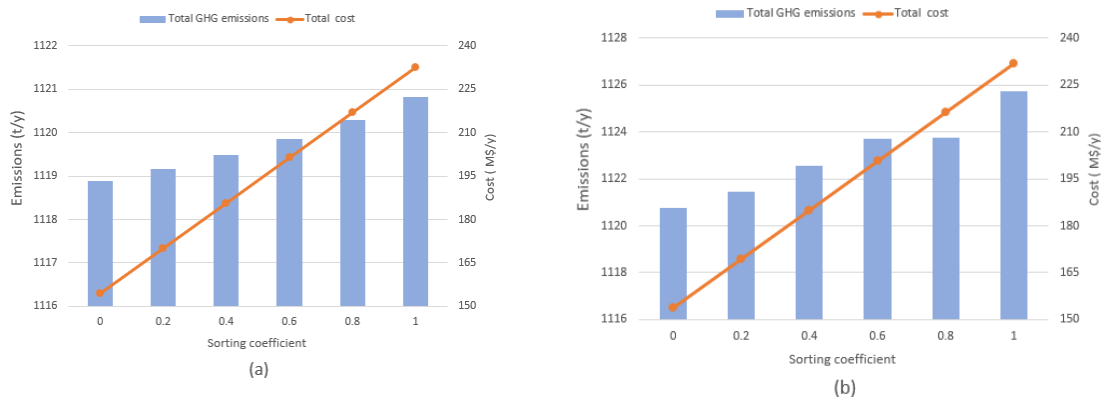


Figure 2: Total cost and GHG emissions (a) Scenario 1 (b) Scenario 2

4.2 Scenario 2: Cost minimization

For the cost minimization scenario, the minimum total cost is 192.83 M\$/y. The costs of MSW transport and sorting process are 6.9 and 39.07 M\$/y. The GHG emissions are 1,122.42 t/y. Compared with Scenario 1, the reduction of the cost is at the cost of the increase of GHG emissions. This indicates a trade-off between total cost and emissions in MSW disposal. Figure 2b indicates the effect of waste sorting coefficient on total cost and GHG emissions under cost minimization.

4.3 Scenario 3: trade-off between total cost and emissions

The cost and GHG emissions of MSW management system are simultaneously considered in this scenario. Fuzzy optimization is used to seek a compromise solution that optimizes the overall level of satisfaction across individual objectives. Solving fuzzy model gives a compromise solution with a total cost of 192.99 M\$/y and an emission level of 1,120.48 t/y. The transport and sorting costs are 6.79 and 39.08 M\$/y. The maximum value of γ is determined to be 0.7585, meaning that each of the cost and emissions objectives is at least 75.85 % satisfied. Figure 3 shows the optimal allocation of MSW for Scenario 3.

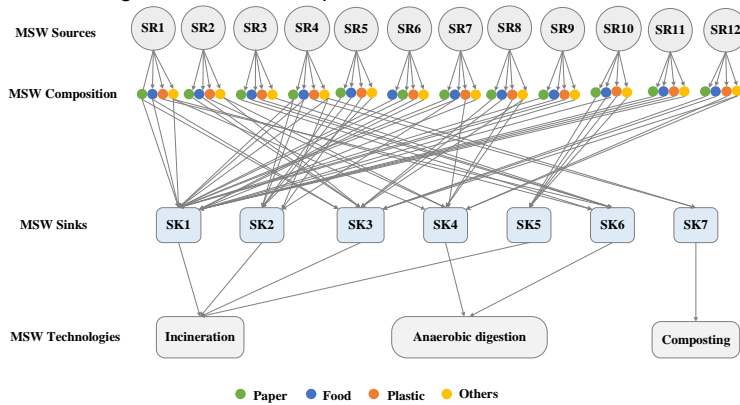


Figure 3: Optimal distribution network of MSW for Scenario 3

5. Conclusions

This paper presents a fuzzy disjunctive optimization approach to tackle the complexities arising from the selection of MSW treatment technologies and the optimization of MSW supply chain. The most suitable treatment technologies and optimal distribution of MSW between sources and sinks are determined with the goal of minimizing both the total cost and GHG emissions. The fuzzy optimization is used to determine compromise solutions with conflicting objectives. The future work should consider the impact of the uncertainty of MSW composition and the capacity of treatment technologies on the vulnerability of MSW supply chain. The carbon tax and the benefit of MSW treatment and the social aspect of MSW treatment needs also to be considered.

Acknowledgments

The work was financially supported by the National Natural Science Foundation of China (Grant No. 52100213), the Fundamental Research Funds for the Central Universities (No. JZ2023HG TB0246), and the Open Project of Key Laboratory of Green Chemical Engineering Process of Ministry of Education (Project No. GCP202109).

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