

# Target-Oriented Robust Optimization of Hybrid Energy Systems under Uncertainty

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Greenhouse gas emissions from the current means of power production are one of the leading contributors to global warming and climate change. The energy market must facilitate the rapid transition to low-carbon and renewable energy sources to replace carbon-emitting and non-renewable fossil fuel sources. However, the utilization of renewable energy sources introduces a new set of challenges in managing operations due to the randomness exhibited by uncontrollable sources. A mix of controllable and uncontrollable sources is required to serve as a buffer in case yield from variable sources becomes insufficient. The benefits of Hybrid Renewable Energy Systems (HRES) rely on the location of the system and the optimal use of energy sources available in the locality to satisfy demand loads at the lowest cost and environmental impact. This study proposes a target-oriented robust optimization model for scheduling the production and distribution of power through a HRES capturing uncertainties in energy source availabilities. An illustrative case study is solved to demonstrate the capabilities of the model. This model provides a portfolio of solutions depending on the risk appetite of the decision-maker. In particular, the results suggest the importance of properly managing the displacement of conventional sources with renewable energy sources, especially when hedging against significant supply and availability variability.

## 1. Introduction

The effects of climate change and global warming are becoming more and more problematic. The greenhouse gases released by the existing methods of power production are one of its major causes. Rapidly switching to low-carbon and renewable energy sources, such as biomass, solar, wind, hydro, and nuclear, in place of carbon-emitting fuel sources, such as coal and natural gas, may help to reduce these emissions.

A transmission grid is often used to distribute power to final users directly. Under this arrangement, the electricity generated must match the power consumed. However, the volatility and randomness of uncontrollable or variable sources, such as solar and wind turbines, present a new set of operational management issues as renewable energy sources' share of the energy distribution networks rises. For instance, solar panels can only generate power when the sun is out, while wind turbines require wind to blow through the turbines. On the other hand, controllable sources, including nuclear, geothermal, and coal power plants, can relatively be turned on and off as needed (Rolnick et al., 2019). A combination of uncontrollable and controllable sources is required to act as a buffer if the yield from variable sources is insufficient due to environmental factors (Yuan et al., 2020). Coal and natural gas power stations are currently used to supply this cushion.

Hybrid energy technologies are receiving more attention because of the inherent stochasticity, discontinuity, and uncontrollability of renewable energies. Several conventional, renewable, or mixed energy sources may be employed in Hybrid Energy Systems (HES) (Ammari et al., 2022). The advantages of HES and their ideal configurations rely on the system's location, the energy sources nearby, and whether they can efficiently fulfill demand loads (Alzahrani et al., 2021). To ensure the effective flow of materials and information and to realize the potential benefits of renewable energy for the environment, society, and the economy, it is crucial to carefully develop, implement, and manage their supply chains and generation networks (Mahjoub and Saheb, 2020).

Power system design, operation, and control issues have frequently been addressed using mathematical optimization techniques. Hermann et al. (2022) performed a comparative evaluation of various optimization

algorithms to design a HES for a building. An artificial neural network-based approach was suggested by Luo et al. (2021) for a bi-level multi-objective optimization of subsidy policy formulation and design for standalone HRES. An integrated optimization approach for capacity sizing and operations planning of a hybrid hydro-photovoltaic HES is proposed by Ming et al. (2021). Rezaei et al. (2021) developed a non-linear optimization model that plans the optimal design and sizing of a hybrid biomass-geothermal system that maximizes energy generation while ensuring minimum pollution and system costs. Tilahun et al. (2021) integrated economic, environmental, and technical goals to maximize the design of a hybrid solar-biomass plant. Most existing models assume that the statistics on energy productivity are deterministic. However, it is incorrect to ignore these uncertainties in the design of such systems (Aliabadi and Radmehr, 2021).

Numerous studies have developed optimization models for hybrid power systems under uncertainty. A popular approach used to optimize similar systems with uncertainty is stochastic programming, such as an island networked hybrid microgrid under wind speed, solar radiation, and load demand uncertainty (Jani et al., 2021), the design and location of a wind-solar hybrid system under generation and load uncertainties (Aliabadi and Radmehr, 2021), the capacity design of a hydro-photovoltaic hybrid system considering flow uncertainties (Li and Yang, 2021). However, rigorous assumptions on the probability distributions of the uncertain parameters, which are particularly difficult to ascertain for complex variables, are required for stochastic programming. This approach typically resolves complex and computationally expensive problems (Pishvaei et al., 2011). Robust optimization came out as a well-recognized alternative as it needs fewer assumptions on the distribution of uncertainty while maintaining the complexity of its deterministic equivalent (Bertsimas and Thiele, 2006). This approach has been used to manage grid-connected energy networks. Robust optimization is critiqued as being too pessimistic because it necessitates strict compliance to all constraints under all realizations of uncertainty leading to the optimization of the worst-case scenario (Guevara et al., 2020). The weaknesses of conventional stochastic programming and robust optimization are addressed by the Target-Oriented Robust Optimization (TORO) approach initially developed by Ng and Sy (2013), which draws from the concept of satisficing targets instead of optimizing while absorbing uncertainties. The decision maker will be presented with an array of solutions that they can select for implementation depending on their risk appetite. TORO has been used in several applications, including polygeneration systems (Aviso et al., 2015) and algal biofuel production (Solis et al., 2021).

This study proposes a Target-Oriented Robust Optimization (TORO) extension to existing models used to schedule the production and distribution of power through a hybrid energy system that captures uncertainties in the availability of renewable energy sources while ensuring that demand loads are efficiently satisfied while achieving the least costs and environmental impact.

Table 1: Sets and their indices

Notation	Definition	Notation	Definition
$i$	Set of energy source/technology, where $i \in I$	$t$	Set of time periods, where $t \in T$

Table 2: Decision and system variables

Notation	Definition	Notation	Definition
$x_{it}$	1 if technology $i$ is operating on time $t$ ; 0 otherwise	$Cost$	System costs
$y_{it}$	Utilized capacity of technology $i$ on time $t$ (MWh)	$Env$	System environmental emissions
$e_t$	Excess power generated on time $t$ (MWh)	$\theta$	Robustness index
$z_t$	1 if power generated on time $t$ satisfies demand; 0 otherwise	$\tau_{Cost}$	Cost targets
		$\tau_{Env}$	Emission targets

Table 3: Parameters

Notation	Definition	Unit
$g_{it}$	Available resources for technology $i$ on time period $t$	MWh
$d_t$	Power demand on time period $t$	MWh
$c_i$	Capacity of technology $i$	MWh
$v_i$	Variable operating costs of technology $i$	US\$/h
$f_i$	Fixed operating costs of technology $i$	US\$
$a_i$	Greenhouse gas emissions generated from operating technology $i$	kg CO <sub>2</sub> -eq
$b_t$	Minimum service level on time period $t$	%
$p$	Selling price of power	US\$/MWh
$m_i$	Minimum level of utilized capacity for technology $i$	MWh

## 2. Model formulation

The Mixed Integer Linear Programming (MILP) model formulation for the HRES problem is given as follows. The model aims to make operational decisions on capacity utilization and energy production that would simultaneously minimize costs and environmental impact. Tables 1 to 3 summarize the nomenclature for the sets, indices, variables, and parameters used in expressing the model's objective function and constraints.

### 2.1 Objective functions

The system aims to minimize overall costs and environmental emissions. Goal Programming was similarly utilized by San Juan et al. (2019) on co-firing problems and, more recently, by Caligan et al. (2022) on water network problems. It is formulated starting with Eq(1) to maximize the achievement of the objectives through an efficiency measure. To balance the efficiency of the conflicting objectives, the minimum between the two efficiencies are maximized. Efficiency is defined as the ratio between actual improvement in the objective value relative the worst performance of the objective and the potential improvement, which is the difference between the best and worst objective values. The best objective value is determined by minimizing each objective function shown in Eqs(2) and (3) as single-objective optimization problems, while the worst objective value is obtained when the other objective is optimized. Costs are incurred from the setup of energy generation technologies each period and from variable operating costs dependent on the capacity utilized. Costs may be offset by revenues earned from selling excess energy produced to the grid. Emissions may be generated from producing energy from various sources or technologies.

$$\text{Max Efficiency} = \text{Min} \left\{ \frac{\text{Cost}_{\text{worst}} - \text{Cost}}{\text{Cost}_{\text{worst}} - \text{Cost}_{\text{best}}}, \frac{\text{Env}_{\text{worst}} - \text{Env}}{\text{Env}_{\text{worst}} - \text{Env}_{\text{best}}} \right\} \quad (1)$$

$$\text{Min Cost} = \sum_i \sum_t (f_i x_{it} + v_i y_{it} - p e_t) \quad (2)$$

$$\text{Min Env} = \sum_i \sum_t a_i y_{it} \quad (3)$$

### 2.2 Constraints

These objectives are subjected to several constraints. Eq(4) limits the utilization of each energy technology by its corresponding installed capacity. Similarly, capacity utilization is limited by the available resources for a particular technology each period as imposed by Eq(5). Eq(6) sets a minimum level of utilized capacity for each technology to ensure that its setup costs are justified. Eq(7) defines service level as the portion of demand that is satisfied by the energy generation technologies operating.

$$y_{it} \leq c_i x_{it} \quad \forall it \quad (4)$$

$$y_{it} \leq g_{it} \quad \forall it \quad (5)$$

$$y_{it} \geq m_i x_{it} \quad \forall it \quad (6)$$

$$\sum_i y_{it} \geq b_t d_t \quad \forall t \quad (7)$$

Eq(8) defines excess power  $e_t$  as the amount of energy produced beyond each period's demand. Eqs(9) and (10) ensure that there can only be excess power sold to market when demand has been fully satisfied. Taking Eqs(7) to (10) together, this implies from the definition of service level that it is permissible for demand to not be fulfilled completely, but in this case, there can be no excess power generated for sale to the grid. The system may only sell excess power when demand has been met fully. Lastly, non-negativity and binary constraints are imposed to the relevant variables.

$$e_t = z_t (\sum_i y_{it} - d_t) \quad \forall t \quad (8)$$

$$\sum_i y_{it} - d_t \leq M z_t \quad \forall t \quad (9)$$

$$d_t - \sum_i y_{it} \leq M(1 - z_t) \quad \forall t \quad (10)$$

### 2.3 Target-Oriented Robust Optimization formulation

The need to incorporate uncertainties originating from the available energy source supply variations into the management of HES has been established. The realizations of these uncertainties during implementation may result in significant negative consequences on the feasibility and performance of the system. Hence, it is vital that the optimal solution is designed to remain feasible even under the most severe degree of uncertainty. The uncertainties in supply will impact Eq(5), and once the deterministic formulation is modified to account for the uncertainty, the revised formulation for the aforementioned constraint is given in Eq(11), where  $\tilde{g}_{it}$  represent uncertain supply parameters.

The uncertainty in supply is handled using the TORO approach developed by Ng and Sy (2013). The model of uncertainty is detailed in Eqs(12) and (13), where  $\bar{g}$  represents the vector of nominal values of each uncertain parameter, while  $\delta g$  contains the perturbations for these uncertain parameters. The largest perturbation occurs

when  $\delta g_k = \widehat{\delta g}_k$ , which is parametrized by the robustness index  $\theta$ . With this methodology, the objective function is replaced with Eq(14), which aims to maximize the robust index ( $\theta \in [0,1]$ ). This is the degree of uncertainty that the solution can tolerate before it becomes infeasible. This objective function is subjected to additional constraints, which are derived from converting the original objective functions into targets through constraints as shown in Eqs(15)-(16). The significance of the robustness index is in its representation of the risk-attitude of a decision maker, specifically a risk-averse decision maker would favor a higher  $\theta$  as this solution is more robust when subjected to uncertainty, while a risk-seeking decision maker would prefer the opposite. The bisection search algorithm is used to maximize  $\theta$ . Multi-objective TORO (MOTORO) approach, recently implemented by Solis and San Juan (2021) and San Juan and Sy (2021), is adopted in this research.

$$y_{it} \leq \tilde{g}_{it} \quad \forall it \quad (11)$$

$$\tilde{g} = \bar{g} + \delta g \quad (12)$$

$$\{\delta g \in \mathfrak{R} \mid 0 \leq \delta g \leq \widehat{\delta g}(\theta)\} \quad (13)$$

$$\text{Max } \theta \quad (14)$$

$$\text{Cost} = \sum_i \sum_t (f_i x_{it} + v_i y_{it} - p e_t) \leq \tau_{\text{Cost}} \quad (15)$$

$$\text{Env} = \sum_i \sum_t a_i y_{it} \leq \tau_{\text{Env}} \quad (16)$$

Setting performance targets is also a critical consideration when implementing the TORO approach. Targets, which are too optimistic, may not be feasible for the system to reach, while pessimistic targets limit the system from achieving better performance results, which may result in significant opportunity costs for the stakeholders. Thus, an approach for identifying appropriate targets is adopted and shown in Eqs(17) and (18).

$$\tau_{\text{Cost}} = \alpha \text{Cost}_{(1)} + (1 - \alpha) \text{Cost}_{(0)} \quad (17)$$

$$\tau_{\text{Env}} = \alpha \text{Env}_{(1)} + (1 - \alpha) \text{Env}_{(0)} \quad (18)$$

Where  $\alpha \in [0,1]$ ,  $\text{Cost}_{(0)}$  and  $\text{Env}_{(0)}$  represent the lowest possible costs and environmental impact under the most favorable conditions of least disruptions where  $\alpha = 0$ , while  $\text{Cost}_{(1)}$  and  $\text{Env}_{(1)}$  assume the highest possible costs and environmental emissions when the highest perturbations occur at  $\alpha = 1$ .

### 3. Computational experiments

In implementing MOTORO, eleven values of  $\alpha \in [0,1]$  were used in increments of 0.1 to identify 11 sets of targets for costs and environmental emissions. For each target, a robust solution maximizing the robustness index is obtained through the bisection search algorithm. The capabilities of the proposed model are demonstrated through an illustrative case study.

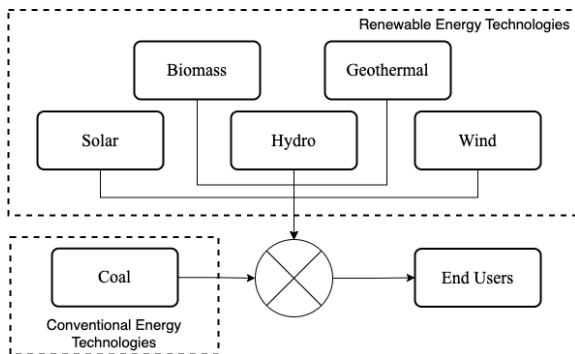


Figure 1: Overview of the hybrid renewable energy system

An HRES composed of 6 energy sources or technologies, which may be broken down into 5 renewable sources and 1 conventional energy technology is considered. These energy sources include biomass, solar, wind, hydro, geothermal, and coal as depicted in Figure 1. This system is studied across the span of a 7-period planning horizon. Hypothetical values were used for the model validation, which are available upon request from the authors. After model implementation, Monte Carlo simulation is performed on the solutions obtained to gauge their respective robustness to uncertainty. Random numbers were generated to represent the uncertainty or perturbations in supply parameters. These instances of uncertainty are used to test whether optimal solutions obtained through TORO can still achieve its cost and environmental impact targets under scenarios different from expected during design. Results are presented in Tables 4 and 5.

Table 4: Hybrid renewable energy system design showing total utilized capacity across 7 periods in MWh

$\theta$	Biomass	Solar	Wind	Hydro	Geothermal	Coal
0.0	1.2786	0.8373	0.8817	0.9662	1.7730	2.0046
0.1 – 0.3	1.2723	0.8253	0.8719	0.9607	1.4812	2.2728
0.4	1.2710	0.8229	0.8699	0.9596	1.4830	2.2733
0.5 – 0.8	1.2496	0.8050	0.8553	0.9514	1.2081	2.5493
0.9	1.2440	0.7786	0.8337	0.9394	1.2192	2.5544
1.0	1.2253	0.7366	0.7995	0.9203	1.2367	2.5708

Table 5: Summary of cost and environmental emissions performance measures

$\theta$	Cost (US\$/h x10 <sup>3</sup> )			Environmental Emissions (kg CO <sub>2</sub> -eq)		
	Targets	Mean	Std. Dev.	Targets	Mean	Std. Dev.
0.0	-415.12	-209.72	8.8131	54,009	53,950	164.1953
0.1 – 0.3	-391.28 – -343.59	-207.63	7.5160	54,284 – 54,834	55,021	140.9780
0.4	-319.75	-206.65	7.2683	55,110	55,038	137.7706
0.5 – 0.8	-295.90 – -224.37	-203.10	5.3846	55,385 – 56,210	56,353	98.6172
0.9	-200.53	-192.86	2.8695	56,486	56,473	52.5550
1.0	-176.69	-176.90	0.0013	56,761	56,757	0.0235

Table 4 summarizes the portfolio of solutions obtained from implementing the MOTORO approach corresponding to different levels of robustness  $\theta$ . Each of these solutions represents a particular configuration and distribution plan of the HRES where the total utilized capacity of each energy technology across 7 periods is indicated. As the robustness index  $\theta$  increases, the system hedges against a higher degree of uncertainty, and availabilities of renewable energy sources are assumed to worsen or go down, while conventional technologies are expected to remain constant. It follows that as  $\theta$  increases, results show that the utilization of renewable energy technologies, namely biomass, solar, wind, hydro, and geothermal power decreases, while coal-based energy generation increases. Under the risk-averse scenario where  $\theta$  approaches 1, the system chooses to err on the side of caution by decreasing dependency on renewable sources in case of supply disruptions. This requires higher utilization of coal to support demand satisfaction. Table 5 presents the system's performance on costs and environmental emissions under varying levels of  $\theta$  together with their corresponding targets. Negative cost values imply that a profit was earned by the system. The results emphasize the tradeoff that exists between the two objectives, while  $\theta$  decreases, signifying risk-seeking behavior, profits increase, and emissions decrease. This follows the earlier discussion. The system has the opportunity, facilitated by improved availability of renewable technologies, to achieve lower environmental emissions but would result in higher investments in renewable energy technologies. Additionally, it is more likely for the system to achieve both of its targets with a more conservative stance at higher levels of  $\theta$ , implying that when renewable energy source availabilities cannot be consistently stabilized, decision-makers who are risk-averse would prefer to maintain reliance on conventional energy despite its environmental consequences.

#### 4. Conclusions

This study proposes a novel multi-objective target-oriented robust optimization model for the design and management of hybrid renewable energy systems under the uncertain availability of renewable energy sources. The model simultaneously optimizes performance on both cost and environmental emissions targets while maximizing robustness or immunity to realizations of uncertainty. The use of the target-oriented robust optimization approach allows for a range of solutions to be presented to the decision-maker to suit varying risk appetites through the robustness index  $\theta$ , which represents the measure of uncertainty the solution can safely and feasibly withstand. These capabilities of the model were presented through an illustrative case study. Results suggest that risk-averse decision makers should still, under volatile renewable energy source availabilities, maintain some dependence on conventional energy sources regardless of environmental impact, as even some utilization of renewable energy can encourage increasing adoption and transition. Future works may explore the consideration of energy storage devices. The model may also be implemented on a large-scale real problem utilizing machine learning techniques to manage bigger data sets.

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