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Optimal Aircraft Payload Weight and Balance using Fuzzy Linear Programming Model

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In recent years, airline operators have not only relied on commercial passenger transport but now depended as well on the income generated by air cargo transport. Inspiring a resurgence and forecast of greater market share for aircraft specific to air freight. To achieve competitive and economic operations, air cargo freighters investigate optimizing many processes within the air cargo loading workflow. One such task is the selection and loading of an aircraft with its designated payload as stakeholders are keen on visualizing the scope of this problem and understanding the tradeoffs between economic objectives, commercial objectives, and design constraints. This is described as the Aircraft Weight and Balance problem, which is a deceptively complex problem that presents itself as a generalized assignment problem. This study presents the development of a fuzzy linear programming model as a decision support tool for air cargo operations in the selection and placement of payload for an optimally transported freight. In contrast with previous studies, which had only considered one or two objectives for optimization, the proposed fuzzy linear programming model allows to maximize the payload, maximize the priority of the package, and minimize the operational cost related to the cargo. This approach provides analysis and visual consideration of more practical operations scenarios in finding solutions that conform to business goals and aircraft regulatory and design limitations. A case study is performed to evaluate the effectiveness of the model. A commercial optimizer engine was used and has been shown to provide solutions to the problem within a short time frame.

1. Introduction

The air cargo sector of the commercial aviation industry is undergoing a transformative age as both the aviation world and the global supply chain adjusts to the new normal post the Covid-19 pandemic. This is attributed to the rapid growth of the e-commerce industry that has blossomed via necessity as a reaction to Covid-19 (Kim, 2020). Signs point toward air cargo as an enticing option for cargo forwarders, as evidenced by a 20 y market forecast that around 2,440 freighter-type aircraft will be introduced in-service, either being newly built or converted from passenger-carrying aircraft (Shparberg and Lange, 2022).

In the context of loading cargo onto aircraft, shipping companies commonly use Unit Loading Devices (ULD) or Pallets to preload cargo before boarding it onto the aircraft. These ULDs/Pallets are containers that have standardized dimensions and weight specifications such that they may be compatible with a variety of aircraft families and their cargo hold variants. In the big picture of cargo operations and logistics, there are many steps across different disciplines in getting a parcel from its origin to its destination i.e., physical tasks of pre-loading and loading the cargo, the actual flight time and aircraft operation, and operational tasks such as load planning, sales, and marketing.

The combination of the challenges and hurdles that arise from the various processes that pertain to air cargo is identified as the Air Cargo Load Planning Problem (ACLPP) (Brandt and Nickel, 2019). One of the main subproblems there is the Aircraft Weight and Balance Problem (WBP), where the goal is to find the optimal set of Cargo to arrange and position in your aircraft while maintaining an appropriate and compliant weight and balance configuration for your aircraft over the various flight regimes. In this problem, the scope exists for the formulation of optimization models.

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The work of Brosh (1981) introduced a fractional programming model that had an objective to maximize profit whilst considering Volume, Weight, and C.G. limits. However, in his study ULDs and Pallets were not used but rather the payload was represented as a bulk load. Further work in incorporating standardized containers like ULDs was done in the work of Mongeau and Bes (2003) where a Mixed Integer Linear Programming (MILP) model was formulated to address the assignment and organization of a set of ULDs to predefined positions and its impact on the Aircraft CG. The objective is to maximize the loaded weight while achieving a target C.G. with a given acceptable displacement ε. Limbourg et al., (2012) introduced a further refined MILP that had the objective of minimizing the moment of inertia difference of the resulting assignment of ULD positions. The work also considered more realistic constraints such as safety load limits and introduced limits for longitudinal and lateral imbalances. It should be noted that this work does consider a defined set of ULDs such that all containers must be loaded and that there may be cases that the aircraft is not fully loaded i.e., whenever the set of total ULDs is less than the total capacity of the aircraft. Vancroonenburg et al., (2014) further contributed to the MILP model of Limbourg and Mongeau such that a bi-objective approach was used through the maximization of profit and minimization of C.G. deviation are now considered. Furthermore, more commercial and both real-world scenarios and constraints have been added to provide better insight into commercial and regulatory needs. The works stated so far mostly consider single-leg flights. Lurkin and Schyns (2015) introduced a model that now considers a two-leg journey wherein only some content of the cargo will be unloaded in the first leg and will be emptied on the next. This work was an extension of Limbourg's model and had an objective to minimize CG offset and overall costs of operations of the two trips (Dahmani and Krichen, 2016) more recently addresses a two-level problem, firstly to assign items into bins (Palletization Problem) and then assigning those bins into loading positions (Weight and Balance Problem). The developed model was a particle swarm optimization approach that had an objective to maximize profit as well as the priority of the items that are a part of the shipment. Zhao et al. (2021) contributed to the previous MILP models upon the consideration of actual C.G. limit envelopes defined by the aircraft manufacturer as constraints whereas previous studies only considered target deviations as constraints. This was a bi-objective model maximizing load as well as minimizing C.G. offset.

There has been some considerable development of the MILP formulations tackling the WBP. However, even with the more recent work considering realistic constraints, there is still a lack of models tackling the practical challenges in commercial operations such as the true nature of planning the shipment based on multiple objectives.

In capturing tradeoffs for conflicting objectives, the true solution would be defined as partially acceptable solutions as was first proposed by Bellman and Zadeh (1970). This concept was then further developed to incorporate multiple objective functions via Linear Programming as introduced by Zimmermann (1978).

In Fuzzy Optimization Models, the solution regions fall in fuzzy sets wherein the elements are partial members. A variable λ is representative of the aggregate degree to which goals are satisfied. Where it is a value between 0 (no membership) and 1 (full membership). There are four functions in which this is represented, Maximizing, Minimizing, Trapezoidal, and Triangular (Zimmermann, 2001). Each with its purpose, when a desired value is high the maximizing fuzzy function should be used, such as for profit and economic gain, for lower desired values the minimizing fuzzy function is used like for cases of minimizing carbon footprint.

In the previous works done, there have either just been 1 or 2 objectives present in the MILP models. Hence, the contribution of this model is to apply fuzzy set theory for the existing MILP models for multiple objective optimizations within the aircraft WBP. The aim of this Fuzzy MILP model would be to provide stakeholders with an additional decision-making tool to visualize the tugging nature of requirements (Payload targets, Aircraft Limitations, Commercial targets, etc.).

The paper is organized as follows; Section 2 provides the problem statement. Section 3 presents the Model Formulation and Methodology. Section 4 shows the results and discussions. Section 5 shows the conclusions.

2. Problem statement

Given a set of 30 containers (i), find the selection set of containers to be loaded onto 9 positions (j) on a Beechcraft 1900 aircraft configured for cargo transport. Each container has a corresponding parameter for Payload Weight (M_i), Priority Score (Pr_i), and Operational Cost Score (C_i). The target is to create a Fuzzy MILP model to select and position 9 containers from the original set of 30 to arrive at an optimal shipset considering Weight goals, Priority goals, and Cost goals with respect to aircraft structural limitations. Note that while there are 30 containers to choose from, an allowance of 2 positions not to be filled can be used if deemed necessary by the loadmaster. this is represented by an additional 2 containers with its mass and priority at 0 and its cost score at 4. The result of this model is then compared to Single Objective Models that focused on (1) Maximizing Payload, (2) Maximizing Priority and (3) Minimizing Operational Cost to understand how the tugging nature of the Fuzzy MILP model was created.

3. Model Formulation and Methodology

3.1 Assumptions and Configuration

The cargo configuration to be analyzed in this model shall be based on the Weight and C.G. calculation exercise performed on a cargo configuration of a Beechcraft 1,900 aircraft from the Federal Aviation Administration (2016) along with hypothetical data to supplement the model. Figure 1 shows the cargo configuration of the Beech 1,900 along with the container set to select cargo from and Table 1 tabulates the respective centroidal moment arm positions and the structural limitations of each position. Aircraft structural limitations relevant to the model are the maximum zero fuel weight (M_{ZFWmax}) of 6,350 kg and longitudinal C.G. envelope of from 7.1 m to 7.6 m. The aircraft also has a Basic Empty Weight (*BOW*) of 4,084.6 kg, *BOW* moment index (Mom_{BOW}) of 29,879 kg-m, and *BOW* C.G. arm (CG_{BOW}) of 7.32 m. Priority and Cost scoring/categories may be defined by the cargo operator; in this model, the assumed categories are listed in Table 2 as categorized by shipment handling codes.

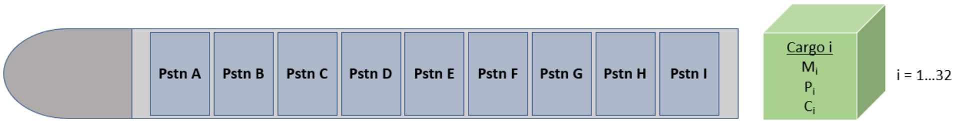

Figure 1: Beech 1900 cargo configuration with 9 loading positions and a set of 30 containers to choose from

Table 1: Cargo Position Station Data

Position	Centroidal Moment Arm (m)	Max. Structural Capacity (kg)
A	5.7	272.2
B	6.5	272.2
C	7.2	272.2
D	8.0	272.2
E	8.8	272.2
F	9.5	272.2
G	10.3	272.2
H	11.0	272.2
I	11.8	272.2

Table 2: Priority and Cost Scoring/Categories by shipment handling codes

Handling Code	Description	Priority Score	Cost Score
SHL	Saving Human Life related Cargo	6	4
LHO	Living Humans Organs/Blood Cargo	5	4
HUM	Human Remains in Coffin Cargo	4	3
DIP	Diplomatic Mail / Diplomatic Cargo	3	1
COM	Company Mail	3	1
MAL	General Mail	1	1
PER	Perishable Cargo	2	2
PEM	Perishable Cargo – Meat	2	2
PEP	Perishable Cargo – Fruits and Vegetables	2	2
PES	Perishable Cargo – Fish and Seafood	2	2
AVI	Live Animal Shipment	2	3
-	General Cargo for everything else	1	1

3.2 Model formulation

The FMILP model for optimizing the WBP has elements of an unbalanced general assignment problem and a 0-1 knapsack problem. We start off the model by defining functions for the total shipment payload weight Eq(1), total shipment priority score Eq(2), and total shipment cost score Eq(3). Where *Xij* is a decision variable if item "*i*" is loaded onto position "*j*" wherein a value of 1 is yes and 0 is no Eq(4). In this problem, *Ncont* = 32 and *Npos* = 9.

$$M_{ptot} := \sum_{i=1}^{Ncont} \sum_{j=1}^{Npos} M_i x_{ij} \quad (1)$$

$$Pr_{ptot} := \sum_{i=1}^{Ncont} \sum_{j=1}^{Npos} Pr_i x_{ij} \quad (2)$$

$$C_{ptot} := \sum_{i=1}^{Ncont} \sum_{j=1}^{Npos} C_i x_{ij} \quad (3)$$

$$x_{ij} \in \{0,1\} \quad (4)$$

An item "*i*" can only be loaded at least once, and a position "*j*" can only be filled once. This is represented in Eq(5) and Eq(6).

$$\sum_{j=1}^{Npos} x_{ij} \leq 1 \;\; for\; all\; i = 1,2,3 \ldots Ncont \quad (5)$$

$$\sum_{i=1}^{Ncont} x_{ij} \leq 1 \;\; for\; all\; j = 1,2,3 \ldots Npos \quad (6)$$

The Aircraft Weight Limitations for maximum zero fuel weight is represented in Eq(7) where the M_{ZFWmax} = 6,350 kg and M_{BOW} = 4,084.6 kg.

$$M_{ZFWmax} \geq M_{BOW} + M_{ptot} \quad (7)$$

For C.G. calculations, the model expresses it in the form of the moment arms. The Total Payload Moment index for the shipment set when positioned in the aircraft is derived in Eq(8) where $StnX_j$ is the centroidal moment arm of position "*j*". This is then used in conjunction with the basic operational empty weight data of the aircraft to get the total moment index of the aircraft at zero fuel weight Eq(9) and then derive the C.G. at *ZFW* Eq(10).

$$Mom_{ptot} := \sum_{i=1}^{Ncont} \sum_{j=1}^{Npos} M_i StnX_j x_{ij} \quad (8)$$

$$Mom_{ZFW} := Mom_{BOW} + Mom_{ptot} \quad (9)$$

$$CG_{ZFW} := \frac{Mom_{ZFW}}{M_{BOW} + M_{ptot}} \quad (10)$$

The upper and lower limit of the C.G. envelope is defined by the aircraft manual, it is best practice to ensure that the C.G. should be in the middle of that envelope as much as possible. The loadmaster shall set a target C.G. as well as the acceptable deviation from that target. This constraint is expressed in Eq(11). For this case, we will consider CG_T as 7.35 m and the acceptable deviation, e, as 0.15 m. However, this equation is non-linear. But can be expressed as two linear inequalities as shown in Eq(12).

$$CG_T - e \leq CG_{ZFW} \leq CG_T + e \quad (11)$$

$$(M_{BOW} + M_{ptot}) * (CG_T - e) \leq (M_{BOW} * \; CG_{BOW}) + Mom_{ptot} \leq (M_{BOW} + M_{ptot}) * (CG_T + e) \quad (12)$$

In defining our multi-objective optimization through fuzzy sets, the objectives (Mass, Priority Score, Cost Score) shall be expressed as constraints within upper and lower limits. Mass and Priority scores are written as Linear Piecewise Fuzzy Maximizing functions Eq(13) and Eq(14) while the Cost score shall be a Linear Piecewise Fuzzy Minimizing function Eq(15). The Fuzzy Membership function λ is a value between 0 and 1 where 0 represents an unacceptable state and 1 represents an acceptable state, this is defined in the model through Eq(16). Where M_{pu} = 2200 kg, M_{pl} = 1400 kg, Pr_{pu} = 32, Pr_{pl} = 7, C_{pu} = 31, C_{pl} = 7.

$$M_{ptot} \geq M_{pl} + \lambda(M_{pu} - M_{pl}) \text{ ; } where \ M_{pl} \leq M_{ptot} \leq M_{pu} \tag{13}$$

$$Pr_{ptot} \geq Pr_{pl} + \lambda(Pr_{pu} - Pr_{pl}) \ where \ Pr_{pl} \leq Pr_{ptot} \leq Pr_{pu} \tag{14}$$

$$C_{ptot} \leq C_{pu} + \lambda(C_{pl} - C_{pu}) \ where \ C_{pl} \leq C_{ptot} \leq C_{pu} \tag{15}$$

$$0 < \lambda < 1 \tag{16}$$

As the fuzzy membership function ties the 3 parameters together, we set the overall objective of the model to maximize the membership function Eq(17) to arrive at a selected shipset that satisfies multiple objectives.

$$Maximize \ \lambda \tag{17}$$

Eq(1) to Eq(17) describe the proposed Fuzzy MILP optimization model for the WBP. The results of this model are then compared to singular objective models that (1) Maximize Payload Mass, (2) Maximize Priority Score and (3) Minimize operational cost.

4. Results and discussion

In the Aircraft WBP, cargo selection and their positioning directly affect aircraft performance, hence the stringent regulatory and aircraft manufacturer's constraints. Along with commercial obligations (profit) and the time-sensitive nature of airline operations, loadmasters responsible for cargo selection must come up with the final shipment set and their positions in a reasonable timeframe that satisfies the defined objectives and constraints. The FMILP model will be implemented using commercial off-the-shelf software, LINGO 20 utilizing a Core i5 3.2 GHz processor with 16 GB Ram and a 64 bit operating system. The comparisons of Model results are shown in Table 3.

Table 3: Summary of Model Results

Parameter	FMILP Model	(1) Maximize Payload Model	(2) Maximize Priority Model	(3) Minimize Cost Model
Runtime (s)	0.25	0.08	0.06	0.05
Payload Mass M_{ptot} (kg)	1,784.8	1,792.6	1,488.3	1,675.3
Priority Score Pr_{ptot}	20	16	32	11
Cost Score C_{ptot}	19	18	31	15
Resulting CG_{ZFW} (m)	7.494	7.496	7.452	7.47
Fuzzy λ	0.481	-	-	-

With the given constraints of the case study performed, the FMILP model is closest to the Payload maximization model, Model 1, with a 0.4 % and 0.03 % difference between the two models for the M_{ptot} and CG_{ZFW} respectively. For a M_{ptot} trade-off of 7.8 kg and a C_{ptot} of 1, you improve the Pr_{ptot} by 4 points.

Between Model 2 and Model 3, it is highlighted how the priority score and operational cost score are shown to be inversely related, which aligns with typically higher handling costs that come when handling priority goods such as medicines and life or death-related cargo. In the FMILP model, it achieved a 46 % difference between its Pr_{ptot} and Model 2's Pr_{ptot}, as well as a 48 % difference between its C_{ptot} and Model 3's C_{ptot} while achieving higher M_{ptot} than both Models. This FMILP result reflects a more practical solution of the Aircraft WBP, where the chosen shipset would not be on the extremities as suggested by Models 2 and Model 3; emphasizing that the extension of the MILP models to use Fuzzy Set theory has allowed the model to explore a region of solutions not previously captured by singular objective models.

The FMILP attained a globally optimized λ value of 0.481 wherein the model had arrived at a configuration that partially satisfies priority and cost requirements along with a 1784.8 kg M_{ptot}, which translates to M_{ZFW} = 5,869.4 kg (M_{BOW} + M_{ptot}) at CG_{ZFW} = 7.494 m which are well within the aircraft limitations of M_{ZFWmax} = 6,350 kg and C.G. envelope from 7.1 m to 7.6 m. The solution runtime was just done at 0.25 seconds, which is a reasonable runtime for a commercial application of the model to explore more potential real-world solutions for the final shipset selection and position. For this case study, the resulting configuration is seen as an overall improvement when compared to its closest single objective model.

5. Conclusions

A Fuzzy MILP model has been developed as an optimization approach for the Aircraft Weight and Balance Problem to consider multiple objectives (Maximizing Payload, Maximizing Priority, and Minimizing Operational

Cost). The model had successfully shown to be an effective decision-making tool to visualize the tugging nature of potentially conflicting requirements and objectives. It was able to provide an optimal and favorable selection of containers to ship that satisfies all objectives when compared against its closest single objective model. This study can be further extended to an earlier process within the aircraft loading process like palletization. Moreover, further study could be a more exhaustive study of the model considering different flight regimes of the configuration or a set of positions of not just one aircraft but a fleet of aircraft.

Nomenclature

BOW – Aircraft Basic Operational Empty Weight
C.G. or CG – Center of Gravity, m
CG_{BOW} – BOW C.G., m
CG_T – C.G. Target, m
CG_{ZFW} – Zero Fuel Weight C.G., m
C_i – Container "i" Operational Cost
C_{pl} – Total Cost Score Lower Limit
C_{ptot} – Total Payload Operational Cost
C_{pu} – Total Cost Score Upper Limit
e – C.G. accepted delta, m
M_i – Container "i" Mass, kg
Mom_{BOW} – Moment index of BOW, kg-m
Mom_{ptot} – Moment index of total payload mass, kg-m
Mom_{zfw} – Moment index of M_{ZFW}, kg-m
M_{pl} – Total Payload Mass Lower Limit, kg
M_{ptot} – Total Payload Mass, kg
M_{pu} – Total Payload Mass Upper Limit, kg
M_{ZFW} – Zero Fuel Weight, kg
M_{ZFWMAX} – Max Zero Fuel Weight, kg
N_{Cont} – Total Number of Containers to select from
N_{pos} – Total Number of Positions to fill
Pr_i – Container "i" Priority Score
Pr_{pl} – Total Priority Score Lower Limit
Pr_{ptot} – Total Payload Priority Score
Pr_{pu} – Total Priority Score Upper Limit
X_{ij} – Binary decision variable if item "i" is loaded to position "j" (1 = assigned, 0 = unassigned)
λ – Fuzzy Membership Function

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