

# Mathematical Modeling of Mass Transfer Phenomena of Sucrose Molecules During Osmotic Dehydration of Star Fruit

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Star fruit (*Averrhoa carambola* L.) is an Asian fruit widely consumed in eastern Peru. The objective of this study was to quantify mass transfer during osmotic dehydration of star fruit using mathematical modeling. Slices of 8 mm thickness were used and the fruit-to-solute ratio was 1:3. The duration of the process ranged from 0 to 420 minutes. The effects of two process variables were evaluated: syrup concentration (50°, 60° and 70° Brix) and dehydration temperature (40°, 50° and 60°C). The experimental data on water and sucrose transfer were fitted to linear, polynomial, exponential and logarithmic mathematical models. The model with the best goodness of fit was selected based on the correlation coefficient  $R^2$ . The results showed that the quadratic polynomial model successfully predicted solute gain and water loss. The diffusion coefficient was quantified using Crank's model with logarithmic fit, and Peleg's mathematical model was fitted to the moisture loss in the fruit. Sucrose diffusion was maximum at 70 °Brix and 60 °C, while water loss was maximum at 70 °Brix and 60 °C. In the Peleg model of water loss, the highest initial water transfer rate was obtained at 50 °Brix and 60 °C, and the lowest equilibrium moisture content was obtained at 70 °Brix and 60 °C. Higher temperatures and sucrose concentrations were observed to lead to higher diffusion rates for water loss and sucrose gain in star fruit.

## 1. Introduction

Fruit dehydration is an ancient food preservation technique that has experienced a resurgence in recent years due to the growing interest in natural, healthy and practical foods (López et al., 2024). According to Giovanelli et al. (2013), the process of removing water content from fruits inhibits the growth of microorganisms and significantly prolongs the shelf life of the food. There are various fruit dehydration techniques, using high temperatures. However, the problem that arises is that most of these methods not only remove water but also some nutrients, and there is also a high energy consumption (Rashid et al., 2024). Osmotic dehydration (OD), a low temperature technique, emerges as an attractive alternative to conventional drying methods, as it allows partial removal of water while largely preserving the sensory, nutritional and functional characteristics of the fruit (Dhiman et al., 2022). OD is a food preservation technique that takes advantage of the principles of osmosis to extract water from products (Kaushal et al., 2016). This process is based on immersing a food in a hypertonic solution (with a higher concentration of solutes than the food), which generates a concentration gradient that expels water from the food into the solution (Goula et al., 2012). This technique has gained great interest in the work of fruits with high moisture content, because it improves the quality of the product in terms of color, flavor and texture, in addition to the low operating temperatures used, which prevents damage to thermolabile products and reduces the energy costs of the process (Nowacka et al., 2018).

Star fruit is a tropical fruit of Asian origin, but it is also widely consumed in the eastern region of Peru. It has a great visual appeal and a sweet and sour taste, and is highly perishable due to its high water content. It is considered an exotic fruit, it has a distinctive star shape in the cross section, and a particular flavor and color. It is mainly used to prepare jams, jellies, desserts, salads, and drinks. Star fruit has a high content of fiber, antioxidant compounds, organic acids, vitamins, and minerals. The fruit has been identified as beneficial in preventing several health problems, including hypoglycemia, hypocholesterolemia, acute liver damage, and diabetes mellitus II. Although this food has great commercial potential, losses of up to 50% of total production can occur in developing countries in Asia and South America due to its high moisture content, which makes it sensitive to microbial deterioration and the absence of adequate post-harvest procedures (Dhara et al., 2023).

Osmotic dehydration is presented as a promising alternative to extend the shelf life of this fruit, by reducing its water activity and concentrating its bioactive compounds.

This work aims to evaluate the effect of different variables of the OD process on the physicochemical properties of star fruit, in order to optimize the operating conditions and obtain a high-quality product. During osmotic dehydration, the rate of water loss and solids gain has been evaluated. Studies carried out on the diffusion kinetics of food products have been explained by Fick's second law, however, there are also other empirical mathematical models that describe the kinetics during osmotic dehydration, as in the case of five-pointed star-shaped star fruit plates (Oliveira et al., 2019).

## 2. Methodology

### 2.1 Sample preparation

The samples were collected directly from fruit trees located in a commercial plantation in the city of Pichanaki, Peru. Ripe fruits were selected, without visible damage, and with morphological characteristics typical of the sweet variety. The collection was carried out during the production season, between January and March 2023. Before being subjected to dehydration, the samples were thoroughly rinsed with drinking water and then aired in the environment.

### 2.2 Dehydration procedure

For this research, star fruit samples were worked with using the osmosis dehydration method. To carry out this process, the fruits were selected, washed and cut into 2 cm slices. These star-shaped slices were then immersed in a hypertonic saccharose solution at concentrations of 70, 60 and 50 °Brix. The process was carried out at a controlled temperature of 60, 50 and 40 °C. The samples were extracted at the following times: 5, 10, 15, 30, 60, 90, 120, 240, 300, 360 and 420 minutes. The syrup was then extracted at each time and the soluble solids were determined with the refractometer. This test was repeated 3 times for each sample.

### 2.3 Mathematical calculations

The initial moisture content of the carambola was determined prior to osmotic dehydration. Post-dehydration, moisture content measurements were taken at the same time points and across all treatment conditions as those for soluble solids. The moisture content was determined using Eq (1), and the solids gain was determined using Eq (2).

$$\text{Moisture (\%)} = \left[ \frac{(\text{initial weight} - \text{final weight})}{\text{initial weight}} \right] \times 100\% \quad (1)$$

$$\text{Solids gained (\%)} = \left[ \frac{(\text{final weight} \times \text{final solids}) - (\text{initial weight} \times \text{initial solids})}{\text{final weigh}} \right] \times 100\% \quad (2)$$

### 2.4 Mathematical modeling of dehydration kinetics

To determine the diffusion coefficients, we employed the Crank model, renowned for its accuracy in predicting diffusion across various geometries. By solving Eq (3) and (4), this model enabled the estimation of the effective solute diffusivity ( $D_{es}$ ), consistent with findings by Oliveira et al. (2019). Additionally, we subjected experimental osmotic dehydration data, including solute gain and moisture loss in carambola, to a comprehensive regression analysis. We evaluated linear, polynomial, logarithmic, and exponential models, selecting the best fit based on the coefficient of determination ( $R^2$ ). For comparison, we also incorporated the Peleg model (Eq 5) to analyze moisture loss kinetics.

$$\text{Ln} \left( \frac{S_t - S_e}{S_0 - S_e} \right) = \text{Ln} \left( \frac{8}{\pi^2} \right) + D_{es} \frac{\pi^2}{4l^2} t \quad (3)$$

$$f = D_{es} \frac{\pi^2}{4l^2} \quad (4)$$

$$\frac{t}{M - M_0} = K_1 + K_2 t \quad (5)$$

## 2.5 Analysis of the effect of osmotic dehydration kinetics in star fruit

Minitab 19 statistical software was used to evaluate the effects of syrup temperature and concentration on moisture loss and soluble solids gain of the fruit. All experiments were performed in triplicate and the results were represented as mean. Statistically significant differences with a p value less than 0.05 were tested by applying ANOVA.

## 3. Results and discussion

### 3.1 Dehydration efficiency

The experimental data of this process were analyzed to evaluate the increase in soluble solids in the fruit as a result of osmosis (Figures 1 a-c), for three temperatures (40, 50 and 60 °C) and three concentrations of sucrose in the solute (50, 60 and 70 °Brix). Furthermore, the evolution of the moisture content of the star fruit samples as a function of dehydration time was also analyzed (Figures 1 d-f).

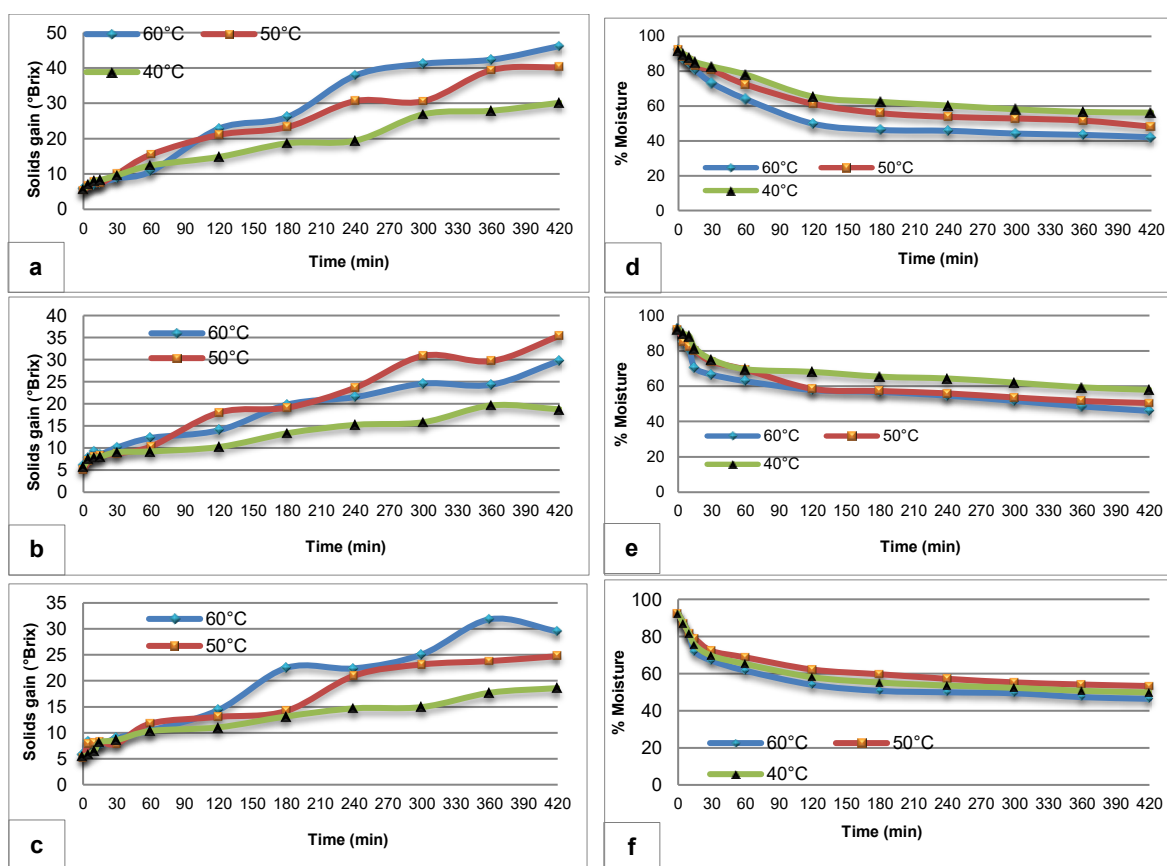


Figure 1 a-f: Data of soluble solids gain (°Brix) in star fruit at 70 °Brix (a). Data of soluble solids gain (°Brix) in star fruit at 60 °Brix (b). Data of soluble solids gain (°Brix) in star fruit at 50 °Brix (c). Data on moisture loss in star fruit at 70 °Brix (d). Data on moisture loss in star fruit at 60 °Brix (e). Data on moisture loss in star fruit at 50 °Brix (f).

The moisture content decreased rapidly and then there was a progressive reduction with increasing osmotic dehydration time. The moisture loss rate was higher at higher temperatures and the solids gain in the fruit was substantially reduced with increasing temperature. According to Tortoe (2009), moisture loss is rapid at the beginning of the process, but becomes increasingly slower during the next stage, as moisture slowly spreads from the inner part to the surface. On the other hand, Giovanelli et al. (2013), reported the appearance of a period of decreasing rate only during the drying of blueberry slices. The hypertonic solution, having a higher concentration of solutes than the inside of the fruit cells, generated a concentration gradient that drove the water out of the star fruit into the solution, resulting in a decrease in moisture content and an increase in soluble solids in the final product.

### 3.2 Mathematical modeling of dehydration

To determine the mathematical model that best fits the experimental data, a semilogarithmic regression ( $\ln(E)$ ) was performed using Crank's equation. Tables 1 and 2 present the results of plotting  $\ln(E)$  as a function of time, for both sucrose gain and water loss in star fruit, respectively. The effective diffusivity ( $D_{es}$ ) was calculated from the slope of these curves. The data were fitted to a linear model, obtaining a higher goodness of fit ( $R^2$ ) with this model. In general, the  $R^2$  values obtained for solids gain show greater goodness of fit than those for water loss.

Table 1: Values  $f$  and Diffusion Coefficient ( $D_{es}$ ) in the diffusivity of soluble solids in star fruit

Concentrations	Temperature	Slope ( $f$ )	Diffusion coefficient ( $D_{es}$ ) $10^{-11}$ m <sup>2</sup> /s	R <sup>2</sup>
70 °Brix	60 °C	-0.00173	18.69713	0.98334
	50 °C	-0.00120	12.96911	0.98147
	40 °C	-0.00100	10.69951	0.94266
60 °Brix	60 °C	-0.00066	7.13301	0.96993
	50 °C	-0.00113	12.21258	0.97804
	40 °C	-0.00017	1.83729	0.72954
50 °Brix	60 °C	-0.00136	14.69832	0.95629
	50 °C	-0.00079	8.53799	0.96459
	40 °C	-0.00025	2.70189	0.71342

Table 2: Values  $f$  and Diffusion Coefficient ( $D_{es}$ ) in water diffusivity of star fruit

Concentrations	Temperature	Slope ( $f$ )	Diffusion coefficient ( $D_{es}$ ) $10^{-11}$ m <sup>2</sup> /s	R <sup>2</sup>
70 °Brix	60 °C	-0.00183	19.77789	0.82450
	50 °C	-0.00123	13.26848	0.89486
	40 °C	-0.00082	8.86222	0.89043
60 °Brix	60 °C	-0.00143	15.45485	0.82442
	50 °C	-0.00123	13.27496	0.85269
	40 °C	-0.00073	7.88954	0.81884
50 °Brix	60 °C	-0.00169	18.26483	0.77877
	50 °C	-0.00116	12.53680	0.83603
	40 °C	-0.00067	7.24108	0.78109

Tables 1 and 2 clearly demonstrate the effect that temperature has on mass transfer during osmotic dehydration. For solutions at 70 °Brix, the diffusivity of soluble solids ( $D_{es}$ ) is found to be directly proportional to temperature. An increase in temperature (from 40 to 60 °C) correlates with steeper slopes, indicating a higher transfer rate of soluble solids.  $D_{es}$  values ranged from 2.70 to 18.69 x 10<sup>-11</sup> m<sup>2</sup>/s, confirming a significant increase due to the combined effects of concentration and temperature. Similarly, effective water diffusivity ( $D_{es}$ ) in star fruit at 70 °Brix is also considerably affected by temperature. The steeper slopes at elevated temperatures (40, 50 and 60 °C) indicate a higher water diffusion rate. These findings confirm that temperature directly influences the kinetics of water diffusion during the process. These results are similar to those worked on by Kaushal et al. (2016), who evaluated the kinetics of osmotic dehydration in jackfruit and reported that the diffusion rate depends on several factors, such as temperature, concentration, particle size, and structural characteristics of the fruit.

### 3.3 Peleg's model

Kinetic models were employed to analyze moisture loss during osmotic dehydration. We initially assessed the Peleg model (Table 3), as well as linear, exponential, and logarithmic functions. Nevertheless, the second-degree polynomial model exhibited the optimal fit to the experimental data, indicated by a coefficient of determination ( $R^2$ ) exceeding 0.9. Concurrently, an increase in process temperature was linked to a more substantial reduction in the product's final moisture content. This effect stems from the heightened molecular

kinetic energy at elevated temperatures, which expedites water diffusion through cell membranes. This result corroborates the findings of Nowacka et al. (2019).

Table 3: Peleg's model parameters for water loss

Concentrations	Temperature	$K_1\left(\frac{\%}{min}\right)$	$K_2\left(\frac{1}{\%}\right)$	$R^2$
70 °Brix	60 °C	-0.98016	-0.01748	0.997
	50 °C	-1.53330	-0.01982	0.993
	40 °C	-2.29044	-0.02220	0.991
60 °Brix	60 °C	-0.72849	-0.02129	0.988
	50 °C	-0.95653	-0.02237	0.996
	40 °C	-1.53667	-0.02665	0.985
50 °Brix	60 °C	-0.65918	-0.02051	0.998
	50 °C	-0.88803	-0.02395	0.997
	40 °C	-2.14643	-0.02715	0.941

To predict the solute gain performance of sucrose syrup during this study, several types of fits were compared: linear, polynomial of degree 2, degree 3, exponential and logarithmic. Among the tested models, the polynomial model of degree 2 provided the best fit for the dehydration kinetics and most accurately predicted system behavior under various conditions (Table 4). With this regression, it is observed that the gain of soluble solids reaches equilibrium as time passes. This, according to Rashid et al., (2024) is due to the fact that, as the concentration of soluble solids in the fruit increases, the concentration gradient decreases and the water flow becomes slower.

Table 4: Modeling and mathematical values and equilibrium time of soluble solids gain

Solute Concentration (°Brix)	Temperature (°C)	Adjusted mathematical model	$R^2$
70	60	$^{\circ}\text{Brix}=5.8+0.15990t-14\times 10^{-5}t^2$	0.988
	50	$^{\circ}\text{Brix}=5.3+0.11967t-9\times 10^{-5}t^2$	0.984
	40	$^{\circ}\text{Brix}=5.7+0.07047t-4\times 10^{-5}t^2$	0.984
60	60	$^{\circ}\text{Brix}=6.1+0.06958t-5\times 10^{-5}t^2$	0.980
	50	$^{\circ}\text{Brix}=5.1+0.08717t-5\times 10^{-5}t^2$	0.983
	40	$^{\circ}\text{Brix}=5.8+0.03720t-2\times 10^{-5}t^2$	0.968
50	60	$^{\circ}\text{Brix}=5.8+0.08808t-7\times 10^{-5}t^2$	0.973
	50	$^{\circ}\text{Brix}=5.5+0.06452t-5\times 10^{-5}t^2$	0.968
	40	$^{\circ}\text{Brix}=5.5+0.04244t-3\times 10^{-5}t^2$	0.967

### 3.4 Effect of different drying kinetics in star fruits

Data processed with Minitab software confirm that both temperature and concentration of the osmotic solution significantly influence star fruit dehydration. In addition, an increase in the gain of soluble solids is observed as both variables increase. This phenomenon is explained by the flow of water from the inside of the fruit into the hypertonic solution, which concentrates the soluble solids in the fruit tissue.

Dhiman et al. (2022) state that osmotic dehydration is essential for preserving nutrients, such as phenolic compounds. They also emphasize that optimizing these processing conditions is essential to extending food shelf life and mitigating lower availability and post-harvest losses. This perspective aligns with Karkou et al. (2021) study on berries and mushrooms. In that study, the use of alternative agents, such as salt and sugar, in osmotic dehydration applied during edible coating resulted in a longer shelf life and improved nutritional value of the products. This approach makes foods available year-round while maintaining their nutritional quality and original form. It can be applied to a wide range of foods, including starfruit.

#### 4. Conclusions

This study investigated the impact of temperature (40, 50, and 60 °C) and osmotic solution concentration (50, 60, and 70 °Brix) on star fruit dehydration. Diffusion coefficients, determined by using the Crank equation with a logarithmic fit, revealed that the highest rates of soluble solids gain and moisture loss were achieved at 70 °Brix and 60 °C. The osmotic dehydration kinetics of star fruit slices were modeled using a second-degree polynomial equation for the evolution of moisture loss, achieving an excellent fit with an  $R^2 = 0.99$ . Similarly, the soluble solids gain was also optimally fitted with a second-degree polynomial equation, obtaining an average  $R^2 = 0.97$ . This study evaluated the effect of sucrose solution concentration and immersion time on dehydration kinetics. The results showed that both variables significantly influenced ( $p$ -value < 0.05) moisture loss and soluble solids gain. Furthermore, a directly proportional relationship was observed between these process variables and the evaluated kinetic parameters (effective diffusion coefficients and rate constant). Samples treated with higher concentration solutions and for longer durations exhibited greater water loss and solids gain, likely due to sustained mass transfer driven by the initial concentration gradient. We recommend that future research focus on independent experimental validation of the developed mathematical models. This is crucial to enable their predictive capacity in new osmotic dehydration scenarios and, thus, ensure their usefulness in industrial-scale process optimization.

#### Nomenclature

$S_t$ - solid concentration at any time $t$	$S_e$ - solid concentration in the product at equilibrium
$S_0$ - initial solid Concentration solids in the product	$l$ - star fruit plate diameter
$K_1$ - Peleg's constant 1	$K_2$ - Peleg's constant 2
$M$ – moisture of star fruit	$M_0$ – initial moisture of strafuit
$f$ - Fick coefficient	$t$ – time of dehydration
$R^2$ - coefficient of determination	

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