

Heat Exchanger Network Synthesis/Retrofit using MINLP Stage-wise Superstructure with Non-isothermal Mixing

Supiluck Koraviyotin^a, Kitipat Siemanond^{b*}

^aThe Petroleum and Petrochemical College, Chulalongkorn University, Soi Chulalong korn 12, Phayathai Rd., Pathumwan, Bangkok 10330, Thailand

^bCenter of Excellence on Petrochemical and Materials Technology, Soi Chulalong korn 12, Phayathai Rd., Pathumwan, Bangkok 10330, Thailand
 kitipat.s@chula.ac.th

One of the major impacts on energy conservation in industrial processes is the integration of heat exchanger network (HEN). Thus, the design of an optimal-cost HEN is a challenging research topic in recently decade. A mixed-integer nonlinear programming (MINLP) stage-wise model by commercial optimization software; GAMS, is developed and allow any split stream flow through multiple exchangers in series as well as bypass stage before mixing non-isothermally for simultaneous synthesis. In the MINLP model where multiple local optima may present, the initialization strategy is therefore developed to systematically find feasible starting point resulting in better HEN design compared to published results in the literatures. In addition, the retrofit of HENs is obtained by adding retrofit constraints to HENS model.

1. Introduction

In the last several decades, the global energy demand is growing dramatically. The industrial sector continues to account for the largest share of delivered energy consumption and is projected to consume more than half of global delivered energy. Therefore, heat exchanger network design takes action for industrial process to enhance the energy recovery by heat integration. To design heat exchanger network (HEN), the methods accomplished with thermodynamic approaches via pinch technology (PT) or mathematical programming (MP) approaches are applied. This work focuses on the mathematical modeling and optimization for HEN design. This work is divided into two main parts, which are, HEN synthesis and retrofit. For synthesis design, in 1990, Yee and Grossmann developed the MINLP model, the SYNHEAT model, based on the stage-wise superstructure for HEN synthesis. To overcome the area trade-off restriction caused by the assumption of isothermal mixing, Huang and Karimi (2012) modified stage-wise superstructure of Yee and Grossman (1990) to allow non-isothermal mixing occurring on the splitting streams of the same process stream. They also improved the bounds of the branch stream temperature by not limit them within their supply and target temperatures thus the better HEN is provided. In contrast to synthesis, Bagajewicz et al. (2013) developed Heat Integration Transportation Model (HIT) for retrofitting HEN of crude units which is mathematical programming-based MILP model to minimize utility cost and capital investment cost simultaneously. The model uses the concept of transportation of heat from a hot stream temperature interval to a cold stream temperature interval where the temperature difference is allowed. All equations in model are linear. This work proposes the modified Synheat model by Yee and Grossmann (1990). The model is MINLP often involving highly non-convex terms; therefore, a new systematic initialization strategy is implied to provide good initial values for simultaneous synthesis and retrofit of HENs. The remaining sections are structured as follows. In section 2, the modified Synheat model (model A1 and A2) without the isothermal mixing assumption will be presented and extended to allow the series of exchanger matches on the same branch flow and also stage-

bypass. The model formulations and effective initialization strategy for both synthesis and retrofit design will be presented in section 3 and 4, respectively. Finally, two examples are solved to demonstrate the significant efficiency of our model.

2. The modified Synheat model

To represent a HEN, a simple stage-wise model was applied due to the low level of non-linearity of the MINLP optimization model. It allows streams entering each stage be able to split up in each stage interval and then are mixing at the end of each interval. However, it did not account for non-isothermal mixing in order to simplify the model with linear heat balances around the exchangers as well as linear heat mixing equations. Parenthetically, the model under assumption of isothermal mixing causes the restriction of the area trade-offs between the exchangers and the overestimation of the area cost.

2.1 Model A1: The modified Synheat model with non-isothermal mixing

In this work the modified model from the stage-wise model (1990); model A1, is used to eliminate the limitation of isothermal mixing assumption. It introduces the non-linear and non-convex constraints accounting for the heat capacity flow rate of each splitting stream and the outlet temperature of each exchanger as presented in Figure 1.

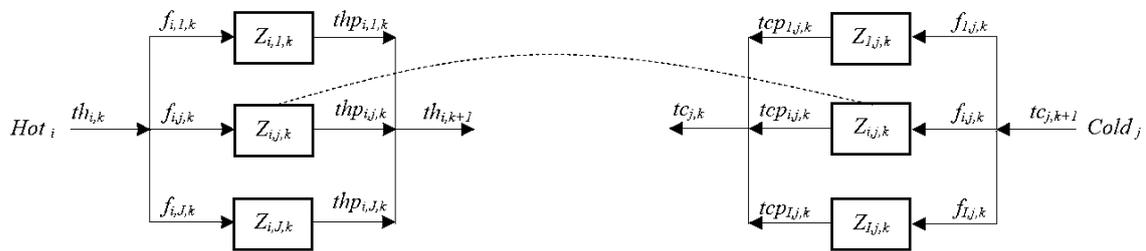


Figure 1: The modified Synheat model (model A1) with non-isothermal mixing

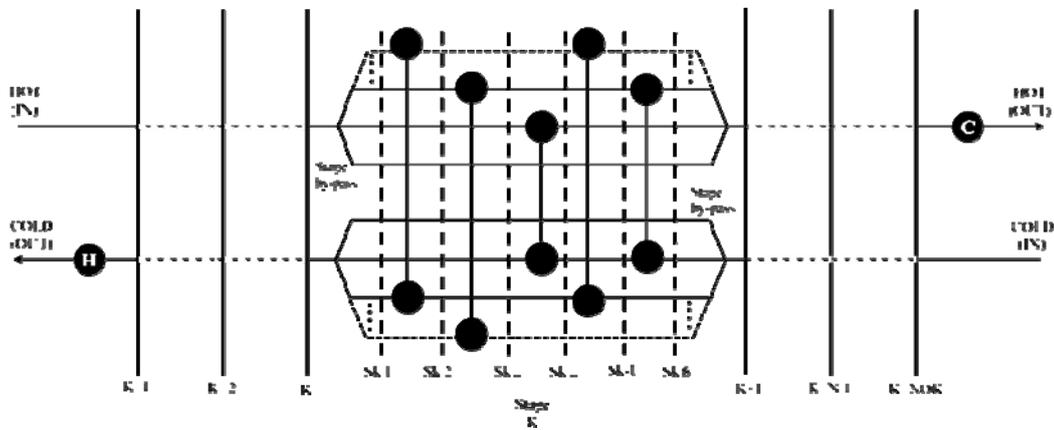


Figure 2: The modified and extended Synheat model (model A2) with non-isothermal mixing allowing several matches per branch of splitting stream

2.2 Model A2: The modified and extended Synheat model

In contrast to the modified model A1, model A2 allows any splitting stream flow through more than one exchanger as depicted in Figure 2. Each of hot or cold streams entering stage K is split into number of sub-streams where each sub-stream is divided into sub-stages, SKs. At each sub-stage SK, any hot splitting stream can exchange heat with any cold splitting stream through several exchanger matches or does not recovery heat through a bypass stage. Thus the model requires extra set of temperature variables in each sub-stage for a hot stream in the hot end of exchanger or for a cold stream in the cold end of exchanger. At last sub-stage; SK of each stage; K, splitting streams merge to form the main stream. To overcome the area trade-off restriction caused by the assumption of isothermal mixing, the outlet temperatures of each splitting stream can be varied at last sub-stage. Finally, the target temperature of each main stream is achieved by using utility at last stage K.

3. Heat Exchanger Network Synthesis

3.1 Model formulation

The reader is referred to the paper by Yee and Grossmann (1990). Some of the new model equations used in model are shown as follows.

Model objective function

$$\begin{aligned} TAC = \{ & [\sum_{i,j,k,bh,bc,sk} Z_{i,j,k,bh,bc,sk} CFHX + \sum_i zcu_i CFCU + \sum_j zhu_j CFHU] + \\ & [\sum_{i,j,k,bh,bc,sk} (a_{i,j,k,bh,bc,sk}) ACHX + \sum_i acu_i ACCU + \sum_j ah_u_j ACHU] + \\ & [\sum_i qcu_i CCU + \sum_j qhu_j CHU] \}; \quad i \in I, j \in J, k \in ST, bh \in BH, bc \in BC, sk \in STSK \end{aligned}$$

1. Overall heat balance for each stream

$$FH_i (TH_{i,IN} - TH_{i,OUT}) = \sum_{k,bh,j,bc,sk} q_{i,j,k,bh,bc,sk} + qcu_i; \quad i \in I, j \in J, k \in ST, bh \in BH, bc \in BC, sk \in STSK$$

$$FC_j (TC_{j,OUT} - TC_{j,IN}) = \sum_{k,bc,i,bh,sk} q_{i,j,k,bh,bc,sk} + qhu_j; \quad i \in I, j \in J, k \in ST, bh \in BH, bc \in BC, sk \in STSK$$

2. Heat balance at each stage k

$$FH_i (th_{i,k} - th_{i,k+1}) = \sum_{bh,j,bc,sk} q_{i,j,k,bh,bc,sk}; \quad i \in I, j \in J, k \in ST, bh \in BH, bc \in BC, sk \in STSK$$

$$FC_j (tc_{j,k} - tc_{j,k+1}) = \sum_{bc,i,bc,sk} q_{i,j,k,bh,bc,sk}; \quad i \in I, j \in J, k \in ST, bh \in BH, bc \in BC, sk \in STSK$$

3. Heat balance at each sub-stage sk

$$\sum_{j,bc} q_{i,j,k,bh,bc,sk} = fhp_{i,k,bh} (thp_{i,k,bh,sk} - thp_{i,k,bh,sk+1}); \quad i \in I, j \in J, k \in ST, bh \in BH, bc \in BC, sk \in STSK$$

$$\sum_{i,bh} q_{i,j,k,bh,bc,sk} = fcp_{j,k,bc} (tcp_{j,k,bc,sk} - tcp_{j,k,bc,sk+1}); \quad i \in I, j \in J, k \in ST, bh \in BH, bc \in BC, sk \in STSK$$

4. Assignments of temperature

$$TH_{i,IN} = th_{i,k}; \quad i \in I, k = ST_{first} \quad th_{i,k} = thp_{i,k,bh,sk}; \quad i \in I, k \in ST, bh \in BH, sk = STSK_{first}$$

$$FH_i th_{i,k} = \sum_{bh} (fhp_{i,k,bh} thp_{i,k,bh,sk}) + (FH_i - \sum_{bh} fhp_{i,k,bh}) th_{i,k}; \quad i \in I, k \in ST, bh \in BH, sk = STSK_{first}$$

$$FH_i th_{i,k+1} = \sum_{bh} (fhp_{i,k,bh} thp_{i,k,bh,sk}) + (FH_i - \sum_{bh} fhp_{i,k,bh}) th_{i,k}; \quad i \in I, k \in ST, bh \in BH, sk = STSK_{last}$$

$$TC_{j,IN} = tc_{j,k}; \quad j \in J, k = ST_{last} \quad tc_{j,k+1} = tcp_{j,k,bc,sk}; \quad j \in J, k \in ST, bc \in BC, sk = STSK_{last}$$

$$FC_j tc_{j,k+1} = \sum_{bc} (fcp_{j,k,bc} tcp_{j,k,bc,sk}) + (FC_j - \sum_{bc} fcp_{j,k,bc}) tc_{j,k+1}; \quad j \in J, k \in ST, bc \in BC, sk = STSK_{last}$$

$$FC_j tc_{j,k} = \sum_{bc} (fcp_{j,k,bc} tcp_{j,k,bc,sk}) + (FC_j - \sum_{bc} fcp_{j,k,bc}) tc_{j,k+1}; \quad j \in J, k \in ST, bc \in BC, sk = STSK_{first}$$

5. Temperature feasibilities

$$th_{i,k} \geq th_{i,k+1} \quad thp_{i,k,bh,sk} \geq thp_{i,k,bh,sk+1}; \quad i \in I, k \in ST, bh \in BH, sk = STSK$$

$$th_{i,k} \geq TH_{i,OUT}; \quad i \in I, k \in ST_{last}$$

$$tc_{j,k} \geq tc_{j,k+1} \quad tcp_{j,k,bc,sk} \geq tcp_{j,k,bc,sk+1}; \quad j \in J, k \in ST, bc \in BC, sk = STSK$$

$$tc_{i,k} \leq TC_{j,OUT}; \quad j \in J, k \in ST_{first}$$

6.1 Maximum matching constraints

$$\sum_{j,bc} Z_{i,j,k,bh,bc,sk} \leq 1; \quad i \in I, j \in J, k \in ST, bh \in BH, bc \in BC, sk \in STSK$$

6.2 Flow constraints

$$0 \leq fhp_{i,k,bh} \leq FH_i$$

$$\sum_{i,bh} Z_{i,j,k,bh,bc,sk} \leq 1; \quad i \in I, j \in J, k \in ST, bh \in BH, bc \in BC, sk \in STSK$$

$$0 \leq fcp_{j,k,bc} \leq FC_j$$

Area equations which the logarithmic mean-temperature difference (LMTD) was approximated according to the Paterson (1984) approximation. Although the modified LMTD of Chen (1987) has been used mostly in the literature because it returns a zero LMTD in case of zero either $dth_{i,j,k,bh,bc,sk}$ or $dtc_{i,j,k,bh,bc,sk}$ its accuracy is less than the Paterson (1984) approximation and its calculated area is underestimate.

3.2 Solution strategy

Although the model A2 looks promising, the problem of the model A2 occurs when it handles the effect of high non-convexities which prevent model from obtaining the feasible solution. This difficulty needs to be overcome by applying the effective initialization strategy. This strategy consists of two main steps; initialization and design steps as shown in figure 3. The design steps consists of four more steps; second, third, fourth, and fifth

steps. The initialization step uses model A1 and four design steps use model A2 to generate HEN. For the first step, model A1, which is simpler to solve by MINLP, generates feasible solution, which will be used as initial values for variables in the next design step using model A2. The initialized variables consist of the binary variables of heat exchanger locations as well as continuous variables of heat load distribution and temperature of each stage. The objective of this first step is to minimize TAC. The second step formulates the NLP model to maximize the heat recovery or minimize the utility consumption by solving flow of splitting stream with fixed binary variables from first step. For the third step, the MILP model is used to generate a topology of HEN with the goal of minimum utility consumption and area cost. To obtain better feasible solution the area of heat exchanger in network and the flow rate of splitting stream are optimized in the fourth step using NLP. The better HEN design is done at the fifth step using MINLP. This MINLP model simultaneously synthesizes effective HEN where the main objective is to minimize total annual cost composing of capital and operational expenses.

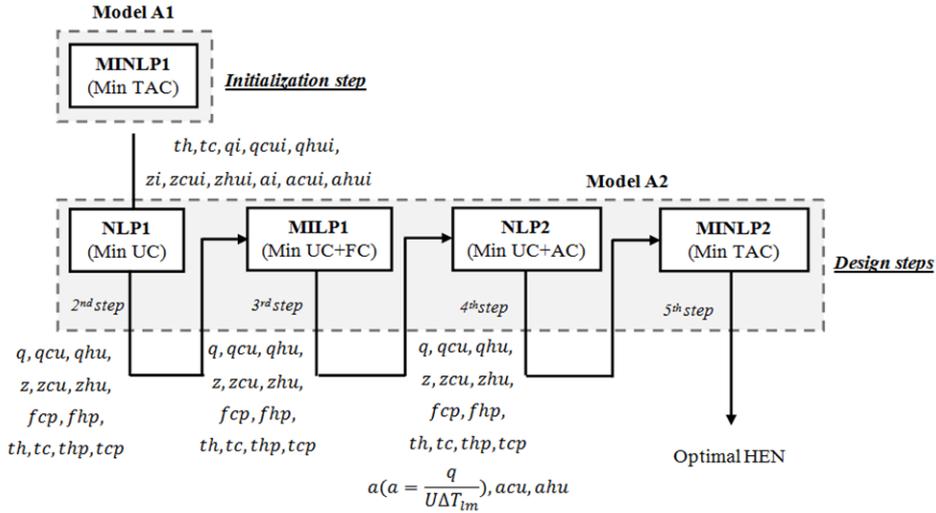


Figure 3: The HEN synthesis strategy

4. HEN retrofit

Different from synthesis design of HEN, retrofit design has no standard problem formulation potentially to revamp any existing networks. It is commonly categorized into two parts, structure and parameter modifications. This work uses the MINLP model based on the modified and extended Synheat model that proposed for HEN synthesis in the section 2.

4.1 Model formulation for HEN retrofit

The aim of the retrofitted HEN with non-isothermal mixing to minimize costs of additional area as well as topology changes involving the addition of exchangers under consideration of using non-profitable exchanger is addressed rigorously;

$$\text{Minimize (Cost)} = \left\{ \left[\sum_{i,j,k,bh,bc,sk} Z_{i,j,k,bh,bc,sk}^{\text{new}} \text{CFHX}^{\text{new}} + \sum_i z_{cu_i}^{\text{new}} \text{CFCU}^{\text{new}} + \sum_j z_{hu_j}^{\text{new}} \text{CFHU}^{\text{new}} \right] + \left[\sum_{i,j,k,bh,bc,sk} (a_{i,j,k,bh,bc,sk}^{\text{add}})^{1/\beta} \text{ACHX}^{\text{add}} + \sum_i acu_i^{\text{add}} \text{ACCU}^{\text{add}} + \sum_j ah_{u_j}^{\text{add}} \text{ACHU}^{\text{add}} \right] + \left[\sum_i q_{cu_i}^{\text{add}} \text{CCU}^{\text{add}} + \sum_j q_{hu_j}^{\text{add}} \text{CHU}^{\text{add}} \right] \right\}; i \in I, j \in J, k \in ST, bh \in BH, bc \in BC, sk \in STSK$$

In addition to HEN synthesis, additional constraints are required in retrofit formulation, for additional areas;

$$\begin{aligned} a_{i,j,k,bh,bc,sk}^{1/\beta} - \text{AHX}^{\text{EXIST}} Z_{i,j,k,bh,bc,sk} &\leq a_{i,j,k,bh,bc,sk}^{\text{add}} \leq \text{AREA}^{\text{max}} - a_{i,j,k,bh,bc,sk}^{1/\beta} + \text{AHX}^{\text{EXIST}} Z_{i,j,k,bh,bc,sk} \\ acu_i^{1/\beta} - \text{ACU}^{\text{EXIST}} z_{cu_i} &\leq acu_i^{\text{add}} \leq \text{AREA}^{\text{max}} - acu_i^{1/\beta} + \text{ACU}^{\text{EXIST}} z_{cu_i} \\ ah_{u_j}^{1/\beta} - \text{AHU}^{\text{EXIST}} z_{hu_j} &\leq ah_{u_j}^{\text{add}} \leq \text{AREA}^{\text{max}} - ah_{u_j}^{1/\beta} + \text{AHU}^{\text{EXIST}} z_{hu_j} \end{aligned}; i \in I, j \in J, k \in ST, bh \in BH, bc \in BC, sk \in STSK$$

Constraints for additional/removal of utility exchangers; heaters and coolers

$$q_{cu_i}^{\text{add}} \leq q_{cu_i} + \text{QCU}^{\text{EXIST}} z_{cu_i}^{\text{EXIST}}; i \in I \quad q_{hu_j}^{\text{add}} \leq q_{hu_j} + \text{QHU}^{\text{EXIST}} z_{hu_j}^{\text{EXIST}}; j \in J$$

Constraints for the new process exchanger/utility exchanger which exchanger location is 0-1 binary variable

$$Z_{i,j,k,bh,bc,sk} = Z_{i,j,k,bh,bc,sk}^{new} + Z_{i,j,k,bh,bc,s}^{EXIST}, \quad Z_{i,j,k,bh,bc,sk}^{new} + Z_{i,j,k,bh,bc,s}^{EXIST} \leq 1$$

$$zcu_i = zcu_i^{new} + zcu_i^{EXIST}, \quad zcu_i^{new} + zcu_i^{EXIST} \leq 1 \quad zhu_j = zhu_j^{new} + zhu_j^{EXIST}, \quad zhu_j^{new} + zhu_j^{EXIST} \leq 1$$

4.2 Solution strategy

The solution strategy for retrofit is similar to one for synthesis. However, it requires more constraints for retrofit steps and the initialization step is to find a feasible solution for minimizing the costs of utility consumptions, additional area and new exchanger installations.

5. Examples

Two examples are given in this section. First example and later one illustrate the proposed model with initialization strategies for synthesis and retrofit problems, respectively. The problems were implemented in GAMS 24.2.1 solved by DICOPT as a MINLP using CONOPT 3 and CPLEX 12.6 as nonlinear programming (NLP) solver and MILP solver, respectively. The input data for example 1 and 2 are summarized in Table 1 and 2, respectively.

Example 1: EMAT= 10 °C, Heat exchanger cost (\$) 6000+600a^{0.85} (a = area in m²)

Example 2: EMAT = 10 °C. Cost of area for a new and existing heat exchanger (\$) 1300a^{0.6} (a = area in m²)

Fixed cost for a new heat exchanger \$3000

Table 1: Stream, cost and solution data for the examples 1

| Stream | T _{in} (°C) | T _{out} (°C) | h (kW ° C ⁻¹ m ⁻²) | F(kW ° C ⁻¹) | Cost(\$ kW ⁻¹ /y) |
|--------|-------------------------|--------------------------|--|-----------------------------|------------------------------------|
| Hot 1 | 155 | 30 | 2.0 | 8 | - |
| Hot 2 | 80 | 40 | 2.0 | 15 | - |
| Hot 3 | 200 | 40 | 2.0 | 15 | - |
| Cold 1 | 20 | 160 | 2.0 | 20 | - |
| Cold 2 | 20 | 100 | 2.0 | 15 | - |
| HU | 220 | 220 | 2.0 | - | 120 |
| CU | 20 | 30 | 2.0 | - | 20 |

Table 2: Stream, cost and solution data for the examples 2

| Stream | T _{in} (°C) | T _{out} (°C) | h (kW ° C ⁻¹ m ⁻²) | F(kW ° C ⁻¹) | Cost(\$ kW ⁻¹ /y) |
|--------|-------------------------|--------------------------|--|-----------------------------|------------------------------------|
| Hot 1 | 170 | 60 | 0.8 | 30 | - |
| Hot 2 | 150 | 30 | 0.8 | 15 | - |
| Cold 1 | 20 | 135 | 0.8 | 20 | - |
| Cold 2 | 80 | 140 | 0.8 | 40 | - |
| HU | 177 | 177 | 0.8 | - | 80 |
| CU | 20 | 40 | 0.8 | - | 20 |

The main objective of Example 1 originally proposed by Björk and Westerlund (2002) is to illustrate the effective HEN synthesis by using proposed model A2 and initialization strategy. The final structure is shown in Figure 4. The network structure obtained in this work is different from all in literatures. The corresponding HEN consists of five heat exchangers with total area of 187.55 m² and TAC of \$94,183, which is less than one in literatures from Björk and Westerlund (2002), and Huang and Karimi (2012) with \$1,818 and \$1,460, respectively. It can be noticed that our proposed model generates the network configuration allowing splitting stream flow through several potential exchangers and trading-off between the number and areas of exchangers affects the optimal TAC. In order to illustrate the retrofit design, Example 2, the example from Yee and Grossmann (1987) is taken. The existing HEN consisting five exchangers requires 1.5×10³ kW steam and 1.9×10³ kW cooling water which the utilities cost is about \$1.58×10⁵/y. The result of HEN retrofit structure is illustrated in Figure 6 requiring one more H1-C2 match of exchanger in the first stage. The modification cost is \$4.36×10⁴ and utilities cost is about \$3.52×10⁴/y from 270 kW steam and 680 kW cooling water. The payback period is about 0.36 y.

6. Conclusion

This work proposed the modified Synheat model that allows non-isothermal mixing and several potential exchangers per branch stream for both synthesis and retrofit problems. To overcome nonlinear-problem-solving difficulty in the MINLP model, it is also very helpful to specify an initial solution for searching starting feasible solution. Two effective steps, initialization and design steps, are proposed to solve simultaneous HENS problem. However, this proposed strategy cannot guarantee that the result is global optimal. The application and usefulness of the proposed model and strategy had been shown by two examples for synthesis and one for retrofit. For the future work, both synthesis and retrofit models will be validated with crude-preheat-train case study simulated by commercial simulation software; PROII. Furthermore, the repiping constraints will be included in the network.

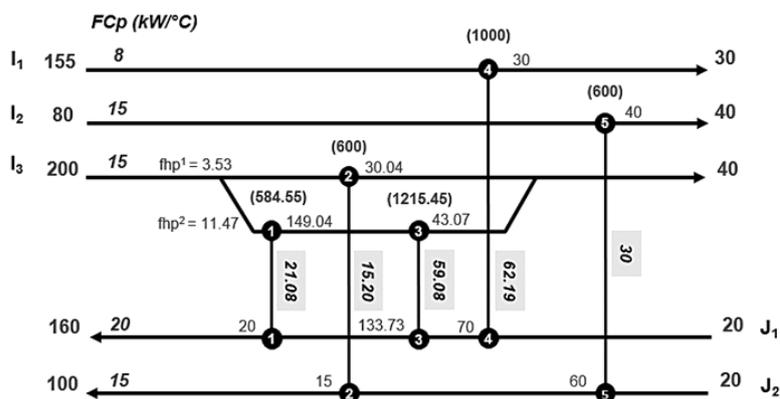


Figure 4: HEN result of example 1

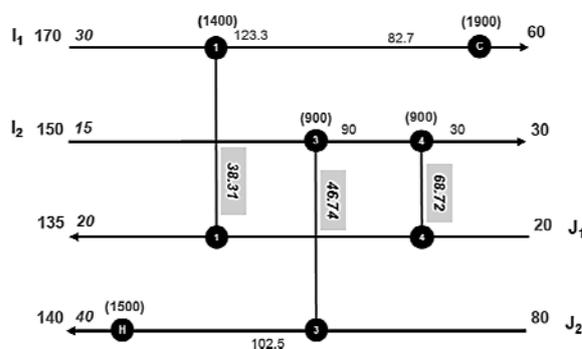


Figure 5: Existing HEN of example 2

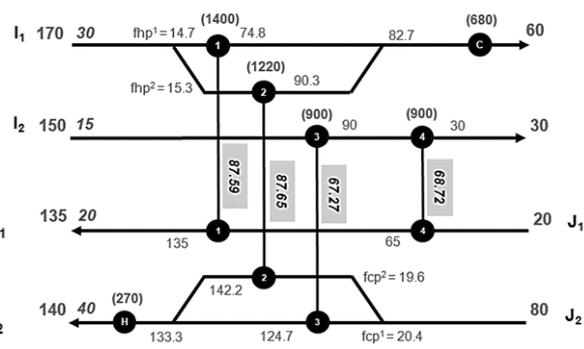


Figure 6: Retrofitted HEN for example 2

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