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A Finite-time Synchronization Scheme for Complex Networks

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This paper presents a method of finite-time synchronization, which can identify the topological structure of complex networks and unknown parameters. According to the theory of the finite-time stability, the feedback controller is designed to realize finite-time synchronization and exponential synchronization. Not only the unknown structure complex network topological is identified but also the parameters of the complex networks are identified. Numerical simulations are presented to illustrate the effectiveness of the theoretical analysis.

1. Introduction

The complex networks consist of the mutual coupling nodes. The types and structure of node of complex networks may be the same or different, which were confirmed (Tung and Chen, 1986; Hoang et al, 1994; Chan and Chau, 1997). Synchronization means that two or more systems, which are either periodic or chaotic, adjust to each other and eventually reach common dynamical behavior, which were confirmed (Zhang and Mao, 2002; Solsona, 1996; Yahyazadeh et al., 2011; Wu et al., 2010; Zhuang, 2007). The synchronization of complex networks may be defined as the node state is consistent with some node or the slave complex network state consistent with the master complex network state, which was confirmed (Mahmoud and Mahmoud, 2010). The exponential synchronization has fast convergence rate which have good effect for synchronization.

Finite-time synchronization has good convergence time which can be calculated in advanced. Finite-time synchronization will be more reasonable than other synchronization due to its synchronization in a given time, finite-time synchronization is more important than asymptotic synchronization. This implies the optimality in settling time. To realize faster convergent time in a control system, finite-time control way is a very important tool. Moreover, the finite-time synchronization with better robustness.

Up to now, few paper research finite-time synchronization, parameter identification and topology weight identification and exponential synchronization in complex dynamical networks. How one realizes these methods in dynamical networks is an urgent issue to tackle. In this paper, we research how to realize them in time delay networks with disturbance.

This paper is organized as follows. In the second section, finite-time synchronization, parameter identification and exponential synchronization between two complex networks with delay and disturbance are researched. In Section 3, we present how to realize finite-time synchronization, parameter identification and exponential synchronization. In Section 4, some numerical simulations are showed to test the effectiveness of theoretical analysis. Finally, some conclusions are drawn in the last part.

2. Preliminaries

A dynamical system with uncertainty can be showed as follows,

$$\dot{x}(t) = f_i(t, x_i(t)) + g_i(t, x_i(t))\alpha_i , \tag{1}$$

Where xi(t)=(x_{i1}(t), x_{i2}(t), …, x_{in}(t)) $^T \in R^n$ is the *i*th node state vector, $\alpha_i \in R^{ni}$ are unknown parameter vectors, i=1, 2, …, N, n_i is nonnegative integers. F_i (t, xi(t)) is an n×1 trix, g_i (t, x_i(t)) is an n×n_i matrix.

The dynamical system with controlled can be described as follows,

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$$\dot{y}(t)= f_i(t, y_i(t))+g_i(t, y_i(t))\hat{\alpha}_i+u_i(t), \tag{2}$$

where $y_i(t)=(y_{i1}(t), y_{i2}(t, \ldots, y_{in}(t))^T \in R^n$ is the *i*th node state vector, $\hat{\alpha}_i \in R^{n_i}$ are unknown parameter estimated vectors, i=1, 2, …, N, n_i is nonnegative integers. $F_i(t, y_i(t))$ is an n×1 matrix, $g_i(t, y_i(t))$ is an $n\times n_i$ matrix.
This paper will use the following definition and lemmas.
Definition 1. The drive－response system are said to be finite-time topology identification and parameter estimated if there is a constant T>0 so that

$$\lim_{t\to T}(\tilde{c}_{ij}-c_{ij})=\lim_{x\to T}(\tilde{\alpha}_i-\alpha_i)=0\text{,} \tag{3}$$

where i,j=1,2,…,N, , topology identification and parameters identification is realize in a finite-time.
Definition 2. The system (2) and the system (1) realize exponentially synchronization if there exists a controller u(t) and positive constants h and α such that the synchronous error satisfies

$$\|e(t)\|\le h\times\exp(-\alpha t),\quad \forall t\ge 0, \tag{4}$$

where α is called the exponential convergence rate.
Definition 3. Supposing that W(t) is a system's Lyapunov function. If W(t) satisfies the following inequality,

$$\dot{W}(t)\le-\lambda W(t)-\beta W^{\omega}(t),\ \ \forall t\ge 0,\ W(0)>0, \tag{5}$$

then the system (5) can realize exponentially finite-time stable, where α,β>0 and 0<ω<1 are constants. In the inequality (5), λ is the exponential convergence rates and β is the finite-time convergence rates.
Lemmas 1, which was confirmed (Chuang et al., 2012)**.** According to **Definition 3**, if the Lyapunov function W(t) satisfies Eq. (5), the following inequality holds:

$$W^{1-\omega}(t)\le-(\beta/\lambda)+\left(\exp\left(\ln(\lambda W(0)^{1-\omega}+\beta)-\alpha(1-\omega)(t-0)\right)/\lambda\right),\ 0\le t\le T,$$
(6)

and w(T)=0 $\forall t\ge T,$ where the finite time T can be illustrated in Eq. (7),

$$T=\left[\ln\left(\left(\lambda W^{1-\omega}(0)/\beta\right)+1\right)/\lambda(1-\omega)\right]\cdot \tag{7}$$

It is clear that finite time T is given by the positive values of α, β and ω. The following **Lemmas 2** extend from the paper (Chuang et al., 2012) claimed is used to proved the main theorem.
Lemmas 2. There exists inequality (8),

$$(a_1+a_2+\ldots+a_n)^c\le a_1^c+a_2^c+\ldots+a_n^c, \tag{8}$$

where $a_1, a_2,\ldots, a_n$ are all positive numbers and 0<c<1

3. Finite-time adaptive synchronization

In this section, we will research finite-time adaptive synchronization.
Now, we consider the complex networks consisting of *N* linearly coupled nodes, every node is n-dimensional dynamical system which can be showed by the following Eq.(9):

$$\dot{x}_i(t)= f_i(x_i(t))+g_i(x_i(t))\alpha_i+\sum_{j=1}^{N}c_{ij}\Gamma x_j(t),\quad i=1,2,\ldots,N, \tag{9}$$

where $xi(t)=(x_{i1}(t), x_{i2}(t), \ldots, x_{in}(t))^T\in R^n$ is the *i*th node state vector, $C=(C_{ij})_{N\times N}\in R$ is an unknown matrix, if node *i* and node *j(i≠j)* with connection, then $C_{ij}\neq 0$. Otherwise, $C_{ij}=0$*(i≠j)*. τ is time delay and τ≥0.
If the model (9) is the drive networks, then the response networks as follows,

$$\dot{y}_i(t)= f_i(y_i(t))+g_i(y_i(t))\hat{\alpha}_i+\sum_{j=1}^{N}\hat{c}_{ij}\Gamma y_j(t)+u_i(t),\quad i=1,2,\ldots,N, \tag{10}$$

Where $y_i(t)=(y_{i1}(t), y_{i2}(t), \ldots, y_{in}(t))^T\in R^n$ is the *I* the node state vector, $u_i(t)$ (i=1,2,…,N) are controllers to be designed, $\hat{C}=(\hat{c}_{ij})_{N\times N}$ is estimated value of weight **C**. $\hat{\alpha}_i$ is the unknown parameter vector α_i, estimated value. T is time delay and τ≥0.

Let $e_i(t)=y_i(t)-x_i(t)$ according to Eq. (9) and Eq. (10), Eq. (11) is the error dynamical network,

$$\dot{e}_i(t)= f_i(t,y_i(t))-f_i(t,x_i(t))+g_i(t,y_i(t))\hat{\alpha}_i-g_i(t,x_i(t))\alpha_i+\sum_{j=1}^{N}\hat{c}_{ij}\Gamma y_i-\sum_{j=1}^{N}c_{ij}\Gamma x_i+u_i(t), \tag{11}$$

The finite-time topology and parameter identification can be regarded as the finite-time stabilization of the error system. The controller $u_i(t)$ can guarantee finite-time synchronization.

Theorem 1. The error system (11) can realize finite-time adaptive synchronization and exponential synchronization via apply the following controller and updated rule,

$$\begin{aligned} u_i(t)=&-f_i(t,y_i(t))-g_i(t,y_i(t))\hat{\alpha}_i-\sum_{j=1}^{N}\hat{c}_{ij}\Gamma y_i+f_i(t,x_i(t))+g_i(t,x_i(t))\hat{\alpha}_i+\sum_{j=1}^{N}\hat{c}_{ij}\Gamma x_i \\ &-\frac{\lambda}{2}e_i(t)-\frac{\beta}{2}e_i^{\frac{p}{q}}(t)-g_i(t,x_i(t))\tilde{\alpha}_i, \end{aligned} \tag{12}$$

$$\dot{\hat{c}}_{ij}=-\frac{\lambda}{2}\tilde{c}_{ij}-\frac{\beta}{2}\tilde{c}^{\frac{p}{q}}-e_i(t)\Gamma x_j(t),\dot{\hat{\alpha}}_i=-\frac{\lambda}{2}\tilde{\alpha}_i-\frac{\beta}{2}\tilde{\alpha}_i^{\frac{p}{q}}, \tag{13}$$

where λ>0, β>0, $\frac{p}{q}\in(0,1)$,p and q are two positive odd integers. The finite-time adaptive synchronization in a finite time

$$T=\left[\ln\left(\left(\lambda W^{1-\omega}(0)/\beta\right)+1\right)\Big/\lambda\left(1-\frac{p}{q}\right)\right], \tag{14}$$

where $W(0)=\frac{1}{2}\sum_{i=1}^{N}e_i^2(0)+\frac{1}{2}\sum_{i=1}^{N}\tilde{\alpha}_i^2(0)+\frac{1}{2}\sum_{i=1}^{N}\sum_{j=1}^{N}\tilde{c}_{ij}^2(0)$, $e_i(0)$, $\alpha_i(0)$, $c_{ij}(0)$ are the initial values of $e_i(t)$, α_i, c_{ij}.

Proof. Lyapunov function as follows,

$$W=\frac{1}{2}\sum_{i=1}^{N}e_i^2+\frac{1}{2}\sum_{i=1}^{N}\tilde{\alpha}_i^2\ \frac{1}{2}\sum_{i=1}^{N}\sum_{j=1}^{N}\tilde{c}_{ij}^2, \tag{15}$$

where $\tilde{\alpha}_i=\hat{\alpha}_i-\alpha_i, \tilde{c}_{ij}=\hat{c}_{ij}-c_{ij}$. $\dot{\tilde{\alpha}}_i=\dot{\hat{\alpha}}_i, \dot{\tilde{c}}_{ij}=\dot{\hat{c}}_{ij}$.

The time derivative of Eq. (15) along the Eq. (11),

$$\begin{aligned} \dot{W}=&\sum_{i=1}^{N}e_i\dot{e}_i+\sum_{i=1}^{N}\tilde{\alpha}_i\dot{\hat{\alpha}}_i+\sum_{i=1}^{N}\sum_{j=1}^{N}\tilde{c}_{ij}\dot{\hat{c}}_{ij} \\ =&\sum_{i=1}^{N}e_i\,[f_i(t,y_i(t))-f_i(t,x_i(t))+g_i(t,y_i(t))\hat{\alpha}_i-g_i(t,x_i(t))\alpha_i+\sum_{j=1}^{N}\hat{c}_{ij}\Gamma y_i-\sum_{j=1}^{N}c_{ij}\Gamma x_i+u_i(t)] \\ &+\sum_{i=1}^{N}\tilde{\alpha}_i[-\frac{\lambda}{2}\tilde{\alpha}_i-\frac{\beta}{2}\tilde{\alpha}_i^{\frac{p}{q}}]+\sum_{i=1}^{N}\sum_{j=1}^{N}\tilde{c}_{ij}[-\frac{\lambda}{2}\tilde{c}_{ij}-\frac{\beta}{2}\tilde{c}_{ij}^{\frac{p}{q}}-e_i(t)\Gamma x_j(t)] \\ =&-\frac{\lambda}{2}[\sum_{i=1}^{N}e_i^2(t)+\sum_{i=1}^{N}\tilde{\alpha}_i^2+\sum_{i=1}^{N}\sum_{j=1}^{N}\tilde{c}_{ij}^2]-\frac{\beta}{2}[\sum_{i=1}^{N}e_i(t)e_i^{\frac{p}{q}}(t)+\sum_{i=1}^{N}\tilde{\alpha}\tilde{\alpha}^{\frac{p}{q}}+\sum_{i=1}^{N}\sum_{j=1}^{N}\tilde{c}_{ij}\tilde{c}_{ij}^{\frac{p}{q}}] \\ =&-\frac{\lambda}{2}[\sum_{i=1}^{N}e_i^2(t)+\sum_{i=1}^{N}\tilde{\alpha}_i^2+\sum_{i=1}^{N}\sum_{j=1}^{N}\tilde{c}_{ij}^2]-\frac{\beta}{2}[\sum_{i=1}^{N}(e_i^2(t))^{\frac{p+q}{2q}}+\sum_{i=1}^{N}(\tilde{\alpha}^2)^{\frac{p+q}{2q}}+\sum_{i=1}^{N}\sum_{j=1}^{N}(\tilde{c}_{ij}^2)^{\frac{p+q}{2q}}] \\ \le&-\frac{\lambda}{2}[\sum_{i=1}^{N}e_i^2(t)+\sum_{i=1}^{N}\tilde{\alpha}_i^2+\sum_{i=1}^{N}\sum_{j=1}^{N}\tilde{c}_{ij}^2]-\frac{\beta}{2}[\sum_{i=1}^{N}e_i^2(t)+\sum_{i=1}^{N}\tilde{\alpha}^2+\sum_{i=1}^{N}\sum_{j=1}^{N}\tilde{c}_{ij}^2]^{\frac{p+q}{2q}} \\ =&-\frac{\lambda}{2}W-\frac{\beta}{2}W^{\frac{p+q}{2q}}. \end{aligned} \tag{16}$$

So, under the controller (12) and updated laws (13), the complex networks (9) and (10) are synchronized in a finite time

$$T = \left[\ln\left(\left(\lambda W^{1-\omega}(0)/\beta \right) + 1 \right) \Big/ \lambda \left(1 - \frac{p}{q}\right) \right].$$

From Eq. (16), we can deduced that

$$\exp(2\lambda t) \times \dot{W}(t) + \exp(2\lambda t) \times 2\lambda W(t) = \frac{d}{dt}[\exp(2\lambda t) \times 2W(t)] \le 0, \quad \forall t < 0. \tag{17}$$

$$\int_0^t \frac{d}{dt}[\exp(2\lambda t) \times W(t)] dt = \exp(2\lambda t) \times W(t) - W(0) \le \int_0^t 0 dt = 0, \quad \forall t > 0. \tag{18}$$

From Eqs. (15) and (18),

$$\begin{aligned} \|e(t)\|^2 \le 2W(t) &= \sum_{i=1}^{N} e_i^2 + \sum_{i=1}^{N} \tilde{\alpha}_i^2 + \sum_{i=1}^{N} \sum_{j=1}^{N} \tilde{c}_{ij}^2 \\ &+ 2 \sum_{i=1}^{N} \int_{t-\tau}^{t} e_i^2(\vartheta) d\vartheta \le 2\exp(-2\lambda t) \times W(0), \quad \forall t \ge 0. \end{aligned} \tag{19}$$

$$\|e(t)\| \le \sqrt{2W(0)} \exp(-\lambda t), \quad \forall t \ge 0. \tag{20}$$

So the controller (12) and updated (13) realize exponential synchronization between two complex networks (9) and (10) in a finite time.

***Remark* 1**. To realize the finite-time stability, the parameters p/q have to satisfy the condition that (p/q) $\in$(0,1) is an irreducible fraction with two positive odd integers p and q. If p is even and q is odd, then the state of $e_i^{\frac{p}{q}}(t)\ (i = 1, 2, \ldots, N)$ is always positive which can cause the system to be unstable when $e_i(t)$ is negative. In contrast, if p is odd and q is even, then the state of $e_i^{\frac{p}{q}}(t)\ (i = 1, 2, \ldots, N)$ will become complex when $e_i(t)$ is negative. The parameter p/q can be assigned arbitrarily under the condition mentioned above, then p+q/2q $\in$(0,1).

4. The simulation research

In this section, taking a chaotic Loenz system is as nodes of network to test the effectiveness of the raised scheme.

The Lorenz system is as follows,

$$\begin{cases} \dot{x}_1 = a(x_2 - x_1) \\ \dot{x}_2 = (b - x_3)x_1 - x_2 \\ \dot{x}_3 = x_1 x_2 - c x_3 \end{cases} ,$$

(21)

where x1, x2, x3 are state variables. When parameters a=10, b=28, c=8/3 the system (21) is chaos.

The coupling configuration matrix $C=(C_{ij})_{N\times N}$ is chosen be as follows

$$\boldsymbol{C} = \begin{bmatrix} -1 & 0 & 1 \\ 1 & -1 & 0 \\ 0 & 1 & -1 \end{bmatrix}.$$

The chaotic Lorenz system is the nodes of networks (1) and (2) which can be described as follows,

$$\begin{pmatrix}\dot{x}_{i1}(t)\\ \dot{x}_{i2}(t)\\ \dot{x}_{i3}(t)\end{pmatrix} = \begin{pmatrix}0\\ -x_{i1}x_{i3}-x_{i2}\\ x_{i1}x_{i2}\end{pmatrix} + \begin{pmatrix} x_{i2}-x_{i1} & 0 & 0\\ 0 & x_{i1} & 0\\ 0 & 0 & -x_{i3}\end{pmatrix}\begin{pmatrix}a\\ b\\ c\end{pmatrix} + \sum_{j=1}^{3} c_{ij}x_j(t-\tau),\quad i=1,2,3,$$

$$\begin{pmatrix}\dot{y}_{i1}(t)\\ \dot{y}_{i2}(t)\\ \dot{y}_{i3}(t)\end{pmatrix} = \begin{pmatrix}0\\ -y_{i1}y_{i3}-y_{i2}\\ y_{i1}y_{i2}\end{pmatrix} + \begin{pmatrix} y_{i2}-y_{i1} & 0 & 0\\ 0 & y_{i1} & 0\\ 0 & 0 & -y_{i3}\end{pmatrix}\begin{pmatrix}\hat{a}_i\\ \hat{b}_i\\ \hat{c}_i\end{pmatrix} + \sum_{j=1}^{3} \hat{c}_{ij}y_j(t-\tau)+u_i(t),\qquad i=1,2,3,$$

where τ is time delay, the controller $u_i(t)$ and updated laws as follows

$$u_i(t) = -f_i(t,y_i(t)) - g_i(t,y_i(t))\hat{\alpha}_i - \sum_{j=1}^{3}\hat{c}_{ij}\Gamma y_i + f_i(t,x_i(t)) + g_i(t,x_i(t))\hat{\alpha}_i + \sum_{j=1}^{3}\hat{c}_{ij}\Gamma x_i$$
$$-\frac{\lambda}{2}e_i(t) - \frac{\beta}{2}e_i^{\frac{p}{q}}(t) - g_i(t,x_i(t))\tilde{\alpha}_i,$$
$$\dot{\hat{c}}_{ij} = -\frac{\lambda}{2}\tilde{c}_{ij} - \frac{\beta}{2}\tilde{c}^{\frac{p}{q}} - e_i(t)\Gamma x_j(t),\ \dot{\hat{\alpha}}_i = -\frac{\lambda}{2}\tilde{\alpha}_i - \frac{\beta}{2}\tilde{\alpha}_i^{\frac{p}{q}},\qquad i=1,2,3.$$

In the numerical simulation, taking the initial states as: $x_i(0)=(1,2,3)^T$, $y_i(0)=(4,5,5)^T$, $\hat{c}_{ij}(0)=1$, $(\hat{a}_i(0),\hat{b}_i(0),\hat{c}_i(0))=(-1+i,1+2i,3+i)$, where 1≤I, j≤3. selecting λ=β=1. Figure 1 shows the synchronization errors, Figure 2 shows the parameters identification, and Figure 3 shows the identification of networks weight.

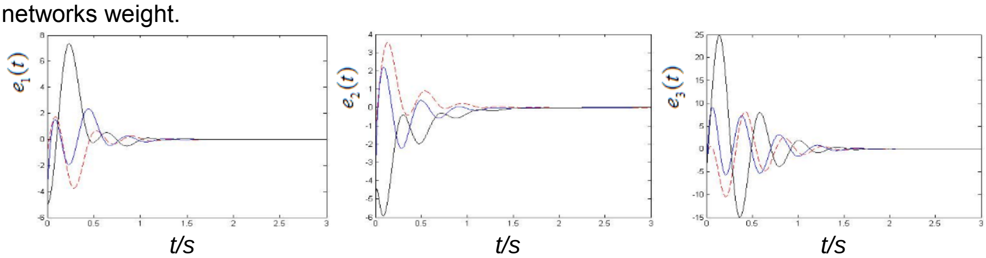

Figure 1: The synchronization errors between two complex networks

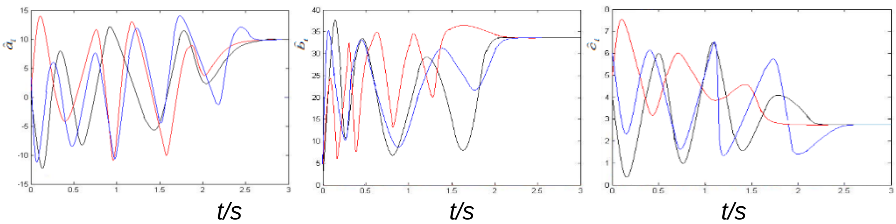

Figure 2: Parameters identification between two complex networks

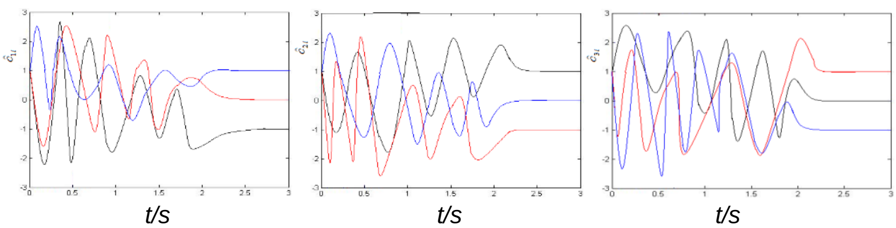

Figure 3: Identification of networks weight between two complex networks

5. Conclusions

This paper presents an approach for complex networks with delay and disturbance to realize finite-time synchronization, parameter identification and exponential synchronization. When complex networks with delay exist parameters disturbance, exotic and topology weight disturbance, based on the theory of finite-time stability and Lyapunov theory, a feedback controller with updated law is designed to not only realize finite-time synchronization and exponential synchronization but also realize parameter and topology identification. Numerical simulation is provided to show the effectiveness of the proposed method.

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