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Research on Probability Density Estimation Method for Random Dynamic Systems

Yanrong Ni*, Ru Min

Department of Electrical Engineering, Henan Institute of Technology
niyanrong666@126.com

In this paper, the author researches on probability density estimation method for random dynamic systems. It indicate that, the Probability Density Function (PDF) curves and failure probabilities of stochastic systems with disjoint failure domains, multiple design points and discontinuous response are calculated effectively and accurately. Moreover, the Probability Density Estimation Method (PDEM) has much higher efficiency, and is a potential and general approach to attack the reliability analysis of complicated problems. Subsequently, the uncertainty propagation and dynamic reliability analysis of nonlinear random system under non-stationary excitations are addressed. The first passage failure criterion and stochastic harmonic function of power spectrum density model of non-stationary random process for earthquake action are utilized. The PDEM can achieve efficiently and accurately the dynamic response, transient PDF and reliability of nonlinear systems.

1. Introduction

Static and dynamic reliability analysis and uncertainty propagation of nonlinear stochastic structure are important research topics for domestic and abroad scholars, which have wide application prospect in engineering practice, and provide a solid theoretical basis for reliability design of the large scale and complex structure. Hong (2013) extends the use of probability density evolution method (PDEM) into the reliability analysis of nonlinear stochastic structures with complex performance functions, and dynamic response and reliability analysis of nonlinear structure under non-stationary excitations. Meanwhile, the influence mechanism of stochastic uncertainty propagation is investigated. In reliability analysis and optimal design of structures, there exist some complex limit state functions with failure domain separation, multiple design points and discontinuous response etc. For such kind of reliability analysis of structures with complex limit state functions, FORM and SORM which are widely used and highly efficient become fail. Despite the numerical simulation methods (such as Monte Carlo simulation, subset simulation and line sampling etc.) can solve this problem, but their computational efficiency is quite low. In addition, there is no accurate and efficient computing approach to address the random dynamic response and reliability analysis of nonlinear structures subject to non-stationary excitations. In recent years, Liu (2013) proposed the probability density evolution method which provides a unified solution framework for random vibration and reliability analysis of nonlinear structures. In this paper, the PDEM is applied to solve the difficult and important research subject of reliability analysis of structures with complicated limit state functions and uncertainty propagation. Actually, PDEM can obtain the probability density function of random structures under static and dynamic load, and is independent of any form of explicit and implicit limit state functions.
This is the so-called Curse of Dimensionality. So, in order to effectively analyses high dimensionality data, it is a pivotal step to reduce their dimensional members. Yang’s (2013) paper is to explore a new feature selection way and propose a feature ranking method to reduce feature’s dimensionalities. In his paper, the principle of reducing feature’s dimensionalities is briefly introduced, and the principal ways of feature dimensionality reduction is reviewed. Moreover, the Probability Density Estimation Method (PDEM) has much higher efficiency than the Monte Carlo simulation, and is a potential and general approach to attack the reliability analysis of complicated problems. a novel feature ranking approach is proposed. A simplified approach is introduced to deal with unsupervised data. At last, the algorithm proposed in his paper is realized by MATLAB, and many

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datasets is used to experiment. A lot of cross-validation and others experimental results demonstrate the validity, feasibility and advantage over others of our approach.

2. The Characteristic of random dynamic systems

The stochastic differential equation and dynamical systems can generate a random dynamic system. This is a hot issue of research in recent years. The most basic and important of the theory of the dynamic system is to reveal the dynamics of the long-term development of the system which properties and the basic dynamic characteristics. Jin (2014) gives a special kind of oscillator, the oscillator is composed of n chain, and the chain dynamics system can be used to describe the Hamilton function. The interaction between heat bath and chain through differential equations (often referred to as the system) is to describe. After a series of derivation, the main use of Ito's formula, get the multiplicative ergodic theorem of the corresponding stochastic dynamic system. Qiu (2014) gives a multiplicative ergodic model needs to meet the conditions. The second section according to the conditions of the first section lists the six special models and illustrates them respectively with multiplicative ergodic theorem.

As in practice the systems have uncertainties, the control problem can not use a simple deterministic model to describe, and it need to combine controlling a system and learning a system uncertainty to an issue. The nature of control is that: on the one hand, the control signal can make the system output towards the desired goal (called control action); However, two hands are contradictory, and the former requires the control signal to change smoothly, while the latter wants to maintain certain amplitude of motivation, so control need trade-off. In Li's (2014) paper, linear, nonlinear stochastic system and multi-model nonlinear stochastic systems are studied. For the unknown parameters with Gaussian white noise stochastic linear systems, using "utility function" it is presented a trade-off of learning and controlling of the control strategy. On the one hand, it can control the system toward the desired output: on the other hand, it can learn the unknown linear parameters. Simulation results show the validation of such approach. With unknown parameters of the non-linear stochastic systems control problem, it is proposed a learning and controlling optimization algorithm. Controller can not only control the output to track the desired output, but also can using RBF networks online learn unknown parameters of nonlinear systems. Yang's (2014) simulation results show superiority of the algorithm. For a class of unknown parameters of nonlinear multi-model switching stochastic systems, it is proposed an algorithm, which use RBF network to learn nonlinear functions online, and Bayes posterior probability to estimate the model, according to the cost function obtain the control signal. By simulation, the algorithm can obtain a switching time of system exactly and can accurately track changes in the system.

3. The probability density estimation method for random dynamic systems

The equation of basic function is as equation (1) as follows:

$$\partial_j (C_{ijkl}\partial_k u_l + e_{kij}\partial_k \varphi) - \rho \ddot{u}_i = 0 \tag{1}$$

Under the linear relationship, basic equation is shown in equation (2):

$$\partial_j (e_{ijkl}\partial_k u_l - \eta_{kij}\partial_k \varphi) = 0 \tag{2}$$

The linear differential equation can be expressed into the following simplified forms:

$$L(\nabla,\omega) f(x,\omega) = 0 \,, L(\nabla,\omega) = T(\nabla) + \omega^2 \rho \mathrm{J} \tag{3}$$

In which,

$$T_{ik}(\nabla) = \partial_j C_{ijkl}\partial_l \;\;, t_i(\nabla) = \partial_j e_{ijk}\partial_k \,, \tau(\nabla) = \partial_i \eta_{ik}\partial_k \tag{4}$$

Consider an infinite situation, we have the equation (5) in the following:

$$\rho(\mathrm{x}) = \rho_0 + \rho_1(\mathrm{x}) \tag{5}$$

Consider the propagation, instead the equation (3) with the following form:

$$C(\mathrm{x}) = C^0 + C^1(\mathrm{x}) \,, e(\mathrm{x}) = e^0 + e^1(\mathrm{x}) ,\;\; \eta(\mathrm{x}) = \eta^0 + \eta^1(\mathrm{x}) \;\;, \tag{6}$$

Then we have equation (7) to (11):

$$C^1 = C - C^0, e^1 = e - e^0,\ \eta^1 = \eta - \eta^0,\ \rho_1 = \rho - \rho_0 \tag{7}$$

The containing inclusions can be simplified into the following integral equation set:

$$f(x,\omega) = f^0(x,\omega) + \int_V \mathcal{S}(x - x')(\mathrm{L}^1 F(\mathrm{y}') + \rho_1\omega^2 \mathbf{g}(R)\mathrm{T}_1 f(\mathrm{y}')]S(\mathrm{y}')\mathrm{dy}' \tag{8}$$

In view of the following relationship

$$\frac{1}{2\pi}\int_{-\infty}^{\infty} e^{-ik_3x_3'}dx_3' = \delta(k_3) \tag{9}$$

Equation (8) can be converted into the following form:

$$f(y,\omega) = f^0(y,\omega) + \int_s S(y - y',\omega)\mathrm{L}^1 F(\mathrm{y}',\omega)\mathrm{d}y' + \rho_1\omega^2 \int_{\mathrm{s}} g(y - y',\omega)\mathbf{J}f(\mathrm{y}',\omega)\mathrm{d}y' \tag{10}$$

In which, S is cylinder cross section, $y = (x_1, x_2)$, and

$$g(y - y',\omega) = \frac{1}{(2\pi)^2}\int_0^{\infty} \bar{k}d\bar{k}\int_0^{2\pi} g(\bar{k},\omega)\exp(-i\mathrm{k}\cdot(\mathrm{y}-\mathrm{y}'))d\phi\ \bar{k} = (k_1, k_2) \tag{11}$$

Suppose k_3=0, $g(\bar{k},\omega)$ can be obtained from Equation (8)
For such kind of material, general form of equation (10) is expressed as following equation (12-14):

$$G_{ik}(\bar{k},\omega) = \frac{1}{\rho_0\omega^2}[\frac{\beta^2}{\bar{k}^2 - \beta^2}\theta_{ik} + \bar{k}_i\bar{k}_k\left(\frac{1}{\bar{k}^2 - \alpha^2} - \frac{1}{\bar{k}^2 - \beta^2}\right) + m_i m_k \frac{\beta_\perp^2}{\bar{k}^2 - \beta_\perp^2}] \tag{12}$$

$$\mathcal{g}_{ik}(\bar{k},\omega) = -\frac{1}{\eta_{11}^0}\frac{1}{\bar{k}^2} + \frac{1}{\rho_0\omega^2}\left(\frac{e_{15}^0}{\eta_{11}^0}\right)^2 \frac{\beta_\perp^2}{\bar{k}^2 - \beta_\perp^2} \tag{13}$$

$$\gamma_i(\bar{k}_i,\omega) = \frac{1}{\rho_0\omega^2}\left(\frac{e_{15}^0}{\eta_{11}^0}\right)^2 \frac{\beta_\perp^2}{\bar{k}^2 - \beta_\perp^2} m_i \tag{14}$$

$$\alpha^2 = \frac{\rho_0\omega^2}{C_{11}^0}, \alpha^2 = \frac{\rho_0\omega^2}{C_{66}^0},\ \beta_\perp^2 = \frac{\rho_0\omega^2}{C_{44}'}, C_{44}' = C_{44}^0 + \frac{(e_{15}^0)^2}{\eta_{11}^0} \tag{15}$$

The first one is the function for random dynamic systems in the form of:

$$\left[\frac{1}{c^2}(\frac{\partial}{\partial t} + \varepsilon)^2 - \Delta\right]\bar{g}(r,t) = \delta(t)\delta^2(\mathrm{r}) \tag{16}$$

$$\left[\frac{1}{c^2}(\frac{\partial}{\partial t} + \varepsilon)^2 - \Delta\right](\frac{\partial}{\partial t} + \varepsilon)^2\bar{h}(r,t) = \delta(t)\delta^2(\mathrm{r}) \tag{17}$$

$$\bar{h}(r,t) = \int_{-\infty}^{\infty} f(t-\tau)\bar{g}(r,\tau)d\tau \tag{18}$$

In which, *f(t)* is the corresponding function defined as the following (19):

$$[\frac{\partial}{\partial t} + \varepsilon]^2 f(t) = \delta(t) \tag{19}$$

After $\bar{g}(r,t)$ is obtained, $\bar{h}(r,t)$ can be easily obtained from Equation (18). So, we have:

$$\bar{g}(k,t)=\frac{1}{2\pi}\int_{-\infty}^{+\infty}\frac{e^{-i\omega t}d\omega}{k^2+(\varepsilon-i\frac{\omega}{c})^2}=c^2\Theta(t)\frac{\sin(ckt)}{ck}e^{-\varepsilon t} \tag{20}$$

$$\bar{g}(r,t)=\frac{1}{(2\pi)^2}\int e^{-i\mathrm{kr}}\bar{g}(\mathrm{k},t)d^2k \tag{21}$$

Via Equation (20), (21) can be converted into:

$$\bar{g}(r,t)=\Theta(t)\frac{c}{(2\pi)^2}\int_0^{2\pi}d\varphi,\int_0^{\infty}\sin(k[ct-kr\cos\varphi])dk \tag{22}$$

In the equation, the following property is adopted:

$$\int_0^{2\pi}\sin(kr\cos\varphi)d\varphi=0 \tag{23}$$

For defining, we normalize it

$$\int_0^{\infty}\sin k\lambda dk=\lim_{\varepsilon\to 0+}\int_0^{\infty}e^{-\varepsilon k}\sin k\lambda dk=\lim_{\varepsilon\to 0+}\mathrm{Re}\frac{1}{\lambda+i\varepsilon} \tag{24}$$

Thus, (22) can be represented into:

$$\bar{g}(r,t)=-\frac{c\Theta(t)}{(2\pi)^2}\mathrm{Re}\int_0^{2\pi}\frac{d\varphi}{r\cos\varphi-ct+i\varepsilon} \tag{25}$$

Thus, Equation (25) can be represented as:

$$\bar{g}(r,t)=-\frac{c\Theta(t)}{(2\pi)^2}\frac{2}{r}\mathrm{Re}\int_0^{2\pi}\frac{e^{i\varphi}d\varphi}{e^{2i\varphi}-2\cosh\phi e^{i\varphi}+1}=-\frac{c\Theta(t)}{(2\pi)^2}\mathrm{Re}\frac{2}{ir}\oint_{|s|=1}\frac{ds}{s^2-2\cosh\phi s+1} \tag{26}$$

Within the unit cycle to solve (26), the following can be obtained:

$$\bar{g}(r,t)=-\frac{c\Theta(t)}{2\pi}\frac{2}{r}\mathrm{Re}\frac{1}{\sinh\phi} \tag{27}$$

$$\bar{g}(r,t)=-\frac{1}{2\pi}\frac{\Theta(t-\frac{r}{c})}{\sqrt{t^2-(\frac{r}{c})^2}} \tag{28}$$

$\bar{h}$ expression can be obtained:

$$\bar{h}(r,t)=\frac{\Theta(t-\frac{r}{c})}{2\pi}\left\{t\ln\left(\frac{ct}{r}+\sqrt{\frac{c^2t^2}{r^2}-1}-\sqrt{t^2-\frac{r^2}{c^2}}\right)\right\} \tag{29}$$

Thus, $\bar{g}(r,\omega)$ must be determined, that is:

$$\bar{g}(r,\omega)=\int_{-\infty}^{\infty}e^{i\omega t}\bar{g}(r,t)dt \tag{30}$$

Put (28) into (30) to obtain:

$$\bar{g}(r,\omega)=\frac{1}{2\pi}\int_{r/c}^{\infty}\frac{e^{i\omega t}dt}{\sqrt{t^2-\frac{r^2}{c^2}}} \tag{31}$$

$$\bar{g}(r,\omega)=\frac{1}{2\pi}\int_{0}^{\infty}e^{i\frac{\omega r}{c}\cosh\phi}d\phi \tag{32}$$

$$H_{0}^{1}(z)=\frac{2}{\pi i}\int_{0}^{\infty}e^{iz\cosh\phi}d\phi \tag{33}$$

As per (33), $\bar{g}(r,\omega)$ of Equation (32) can be obtained:

$$\bar{g}(r,\omega)=\frac{i}{4}H_{0}^{1}(\frac{\omega r}{c}) \tag{34}$$

The function component defined for Fourier transform can be obtained in the following equation (35)-(39).

$$G_{ik}(r,\omega)=\frac{i}{4\rho_{0}\omega^{2}}\{\theta_{ik}\beta^{2}H_{0}^{1}(\beta r)-\frac{\partial^{2}}{\partial y_{i}\partial y_{k}}[H_{0}^{1}(qr)]_{\beta}^{\alpha}+m_{i}m_{k}\beta_{\perp}^{2}H_{0}^{1}(\beta_{\perp}r)\}$$

$$\gamma_{i}(r,\omega)=\frac{i}{4\rho_{0}\omega^{2}}(\frac{e_{15}^{0}}{\eta_{11}^{0}})\beta_{\perp}^{2}H_{0}^{1}(\beta_{\perp}r)m_{i} \tag{35}$$

$$[f(qr)]_{\beta}^{\alpha}=f(\alpha r)-f(\beta r)\;\; r=|y| \tag{36}$$

$$\gamma_{i}(r,t)=\frac{e_{15}^{0}}{\eta_{11}^{0}C_{44}'}m_{i}\bar{g}_{3}(r,t) \tag{37}$$

In which, $\bar{g}_{i}$ represents output of Equation (38)

$$\bar{g}_{i}(r,t)=\frac{1}{2\pi}\frac{\Theta(t-\frac{r}{c_{i}})}{\sqrt{t^{2}-(\frac{r}{c_{i}})^{2}}} \tag{38}$$

Meanwhile, it also represents output for equation (29)

$$\bar{h}_{i}(r,t)=\frac{\Theta(t-\frac{r}{c_{i}})}{2\pi}\left\{t\ln\left(\frac{c_{i}t}{r}+\sqrt{\frac{c_{i}^{2}t^{2}}{r^{2}}-1}\right)-\sqrt{t^{2}-\frac{r^{2}}{c_{i}^{2}}}\right\} \tag{39}$$

The experiment result of probability density evolution method (PDEM) and the Gaussian Kernel Parzen Estimation (GKPE) algorithm is shown in the figure 1. From the experiment, we can get that the PDEM can achieve efficiently and accurately the dynamic response, transient PDF and reliability of nonlinear systems.

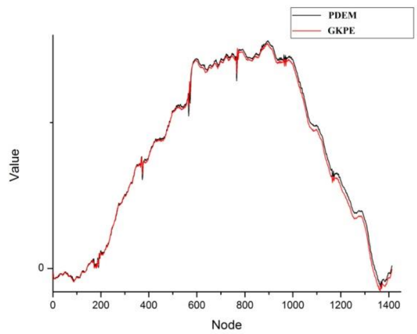

Figure 1: The experiment result of probability density evolution method (PDEM) and the Gaussian Kernel Parzen Estimation (GKPE) algorithm

4. Conclusion

In this paper, the author researches on probability density estimation method for random dynamic systems. As in practice the systems have uncertainties, the control problem cannot use a simple deterministic model to describe, and it need to combine controlling a system and learning a system uncertainty to an issue. The nature of control is that: on the one hand, the control signal can make the system output towards the desired goal (called control action); the other hand, the control signal can reduce the uncertainty of system parameters (called learning the parameter uncertainty). The PDEM is applied to solve the difficult and important research subject of reliability analysis of structures with complicated limit state functions and uncertainty propagation. Actually, PDEM can obtain the probability density function of random structures under static and dynamic load.

References

Hong X., Chen S., Qatawneh A., Daqrouq K., Sheikh M., Morfeq A., 2013, Sparse probability density function estimation using the minimum integrated square error, Neurocomputing, 115-128.

Jin X.L., Wang Y., Huang Z.L., Paola M.D., 2014, Constructing transient response probability density of non-linear system through complex fractional moments, International Journal of Non-Linear Mechanics, 65-76.

Li B., Pang F.W., 2014, Improved cardinalized probability hypothesis density filtering algorithm, Applied Soft Computing Journal, 24-39.

Liu Y., Chen L.Y., Yi H., 2013, Submarine pressure hull butt weld fatigue life reliability prediction method, Marine Systems, 29-29.

Pei Y.J., Evatt R., 2013, Hawkes, Sanghoon Kook. Transported probability density function modelling of the vapour phase of an n -heptane jet at diesel engine conditions, Proceedings of the Combustion Institute, 342-356.

Qiu X., Huang Y.X., Zhou Q., Sun C., 2014, Scaling of maximum probability density function of velocity increments in turbulent Rayleigh-Bénard convection, Journal of Hydrodynamics, Ser. B, 263-278.

Yang Q.S., Tian Y.J., 2014, A model of probability density function of non-Gaussian wind pressure with multiple samples", Journal of Wind Engineering & Industrial Aerodynamics, 541-549.

Yang Y., Wang H.F., Pope S.B., Chen J.H., 2013, Large-eddy simulation/probability density function modeling of a non-premixed CO/H 2 temporally evolving jet flame, Proceedings of the Combustion Institute, 341-358.

Zhu J.Y., Gui W.H., Yang C.H., Xu H.L., Wang X.L., 2014, Probability density function of bubble size based reagent dosage predictive control for copper roughing flotation, Control Engineering Practice, 128-142.